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RESEARCH ARTICLE

IQ Index Interpretation, Using Fuzzy Sets

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Abstract

Intelligence quotient IQ is a number claiming to measure the intelligence of a human been. This number is established based on psychometric testing. In use in the English-speaking world, there is a variety of individually administered IQ tests, having different degree of difficulty. In this paper, we propose a computational method of interpretation of IQ numbers taking into account on the 'exigency', of the person who make the interpretation. For this purpose, to different interpretations, having different 'exigency degree', different fuzzy sets are associated. This attachment makes the interpretation of IQ tests (Which claims to measure the natural world 'intelligence') computational comparable. Computations are presented and significant differences are revealed for example concerning computational identification of group of persons having intelligence in a given range. Understanding of fuzzy logic concepts, of operations with fuzzy sets and understanding of fuzzy logic operators are presented in this context.

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The Fuzzy Language Expression 'Human Intelligence'

Expressing actions and attributes of human been at the same time, faber and sapiens, the natural language expression human intelligence could not (Even after it became an object of science) benefit from a classical definition, through delimitations of proximate gender and specific difference. In natural language, human intelligence is the intellectual capability of humans, which is marked by complex cognitive feats and high levels of motivation and self-awareness. Using their intelligence, humans are able to learn, form concepts, understand, and apply logic and reason. Human intelligence is also thought to encompass their capacities to recognize patterns, plan, innovate, solve problems, make decisions, retain information, and use language to communicate. The term human intelligence has been present

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since time immemorial in natural language, enshrined in literature (Apparently by Cicero) and characterizes (From various angles) the power and function of the human mind to establish connections and make connections between connections: it is what suggests inter-legere, bringing together two meanings - to discriminate between and to bind (To gather, to put together). Of all human abilities, the most specifically human characteristic is intelligence, given that it transforms biological man into Homo Sapiens. However, intelligence is not a material thing, but an abstract concept, being difficult to define. We can say that one analyzes the manifestations of intelligence, the faculties that define intelligence, but not intelligence itself. Intelligence has been defined and studied by each psychological school according to the general postulates of the conception of Man. The psychology of human intelligence is closely related to the concept of individual differences in mental "traits" and the development of analytical tools. Throughout history, the concept of human intelligence has changed a lot. The evolution of human intelligence refers to several theories that seek to describe how human intelligence evolved in relation to the evolution of the human brain and the origin of language [1]. The timeline of human evolution spans about 7 million years, from the separation of the genus Pan to the emergence of behavioral modernity 50,000 years ago. Of this timeline, the first 3 million years concern Sahelanthropus, the next 2 million concern Australopithecus, while the last 2 million cover the history of Homo reale (Paleolithic) species. Many features of human intelligence, such as empathy, mourning, ritual, and the use of symbols and tools, are already evident in the great apes, albeit at a less sophisticated level than in humans. There is a debate between proponents of the idea of a sudden emergence of intelligence, called the "Great Leap Forward" and proponents of a "Gradual Emergence" (Continuous) hypothesis of intelligence.

Theories of the evolution of human intelligence include: Robin Dunbar's Social Brain Hypothesis [2], Geoffrey Miller's sexual selection hypothesis (concerning sexual selection in human evolution) [3], The hypothesis called Ecological Dominance-Social Competition (EDSC) [4] (explained by Mark V. Flinn, David C. Geary, and Carol V. Ward, based primarily on the work of Richard D. Alexander), The intelligence hypothesis as a signal of good health and disease resistance, The hypothesis called group selection theory (this holds that organismal characteristics that benefit a group (Clan, tribe, or larger population) can evolve despite individual disadvantages, such as those cited above), The hypothesis that intelligence is connected to nutrition and thus to status [5]. (This supports the idea that a higher IQ in a person could be a signal that the person comes from and lives in a physical and social environment where nutrition levels are high and vice versa).

Theories of the human intelligence include:

-Multiple intelligences theory of Howard Gardner. This theory is based on studies of normal children and adults, of gifted individuals (including so-called "savants"), of persons who have suffered brain damage, of experts and virtuosos, and of individuals from diverse cultures. Gardner breaks intelligence down into components. In the first edition of his book Frames of Mind (1983), he described seven distinct types of intelligence: logical-mathematical, linguistic, spatial, musical, kinesthetic, interpersonal, and intrapersonal. In a second edition, he added two more types of intelligence: naturalist and existential intelligences. He argues that psychometric (IQ) tests address only linguistic and logical plus some aspects of spatial intelligence [6]. A criticism of Gardner's theory is that it has never been tested, or subjected to peer review, by Gardner or anyone else, and indeed that it is unfalsifiable [7]. Others (e.g. Locke,2005[8]) suggest that recognizing many specific forms of intelligence (specific aptitude theory) implies a political-rather than scientific-agenda, intended to appreciate the uniqueness in all individuals, rather than recognizing potentially true and meaningful differences in individual capacities. Schmidt and Hunter [9] suggest that the predictive validity of specific aptitudes over and above that of general mental ability, or "g", has not received empirical support. On the other hand, Jerome Bruner agreed with Gardner that the intelligences were "useful fictions", and went on to state that "his approach is so far beyond the data-crunching of mental testers that it deserves to be cheered" [10].

Robert Sternberg proposed the triarchic theory of intelligence to provide a more comprehensive description of intellectual competence than traditional differential or cognitive theories of human ability [11]. The triarchic theory describes three fundamental aspects of intelligence:

1. Analytic intelligence comprises the mental processes through which intelligence is expressed.

2. Creative intelligence is necessary when an individual is confronted with a challenge that is nearly, but not entirely, novel or when an individual is engaged in automatizing the performance of a task.

3. Practical intelligence is bound to a sociocultural milieu and involves adaptation to, selection of, and shaping of the environment to maximize fit in the context.

The triarchic theory does not argue against the validity of a general intelligence factor; instead, the theory posits that general intelligence is part of analytic intelligence, and only by considering all three aspects of intelligence can the full range of intellectual functioning be understood. Sternberg updated the triarchic theory and renamed it to the Theory of Successful Intelligence [12]. He now defines intelligence as an individual's assessment of success in life by the individual's own (Idiographic) standards and within the individual's sociocultural context. Success is achieved by using combinations of analytical, creative, and practical intelligence. The three aspects of intelligence are referred to as processing skills. The processing skills are applied to the pursuit of success through what were the three elements of practical intelligence: adapting to, shaping of, and selecting of one's environments. The mechanisms that employ the processing skills to achieve success include utilizing one's strengths and compensating or correcting for one's weaknesses. Sternberg's theories and research on intelligence remain contentious within the scientific community [13].

Based on A. R. Luria's (1966) seminal work on the modularization of brain function [14], and supported by decades of neuroimaging research, the PASS Theory of Intelligence (Planning/Attention/Simultaneous/Successive) proposes that cognition is organized in three systems and the following four processes [15]:

1. Planning involves executive functions responsible for controlling and organizing behavior, selecting and constructing strategies, and monitoring performance.

2. Attention is responsible for maintaining arousal levels and alertness, and ensuring focus on relevant stimuli.

3. Simultaneous processing is engaged when the relationship between items and their integration into whole units of information is required. Examples of this include recognizing figures, such as a triangle within a circle *vs.*, a circle within a triangle, or the difference between "he had a shower before breakfast" and "he had breakfast before a shower."

4. Successive processing is required for

organizing separate items in a sequence such as remembering a sequence of words or actions exactly in the order in which they had just Triarchic theory of intelligence PASS theory of intelligence been presented.

These four processes are functions of four areas of the brain. Planning is broadly located in the front part of our brains, the frontal lobe. Attention and arousal are combined functions of the frontal lobe and the lower parts of the cortex, although the parietal lobes are also involved in attention as well. Simultaneous processing and Successive processing occur in the posterior region or the back of the brain. Simultaneous processing is broadly associated with the occipital and the parietal lobes while Successive processing is broadly associated with the frontal-temporal lobes. The PASS theory is heavily indebted both to Luria [14,16] and to studies in cognitive psychology involved in promoting a better look at intelligence [17].

Piaget's theory and Neo-Piagetian theories. In Piaget's theory of cognitive development, the focus is not on mental abilities but rather on a child's mental models of the world. As a child develops, the child creates increasingly more accurate models of the world that enable the child to interact with the world more effectively. One example is object permanence with which the child develops a model in which objects continue to exist even when they cannot be seen, heard, or touched. Piaget's theory described four main stages and many sub-stages in the development. These four main stages are: 1. Sensorimotor stage (birth–2 years) 2. Pre-operational stage (2-7 years) 3. Concrete operational stage (7-11 years) 4. Formal operations stage (11-16 years) [18]. Progress through these stages is correlated with, but not identical to psychometric IQ [19]. Piaget conceptualizes intelligence as an activity more than as a capacity.

One of Piaget's most famous studies focused purely on the discriminative abilities of children between the ages of two and a half years old, and four and a half years old. He began the study by taking children of different ages and placing two lines of sweets, one with the sweets in a line spread further apart, and one with the same number of sweets in a line placed more closely together. He found that, "Children between 2 years, 6 months old and 3 years, 2 months old correctly discriminate the relative number of objects in two rows; between 3 years, 2 months and 4 years, 6 months they indicate a longer row with fewer objects to have 'more'; after 4 years, 6 months they again discriminate correctly" [20]. Initially younger children were not studied, because if at the age of four years a child could not conserve quantity, then a younger child presumably could not either. The results show however that children that are younger than three years and two months have quantity conservation, but as they get older, they lose this quality, and do not recover it until four and a half years old. This attribute may be lost temporarily because of an overdependence on perceptual strategies, which correlates more candy with a longer line of candy, or because of the inability for a four-year-old to reverse situations.

This experiment demonstrated several results. First, younger children have a discriminative ability that shows the logical capacity for cognitive operations exists earlier than previously acknowledged. Also, young children can be equipped with certain qualities for cognitive operations, depending on how logical the structure of the task is. Research also shows that children develop explicit understanding at age five and as a result, the child will count the sweets to decide which has more. Finally, the study found that overall quantity conservation is not a basic characteristic of humans' native inheritance [18]. Piaget's theory has been criticized on the grounds that the age of appearance of a new model of the world, such as object permanence, is dependent on how the testing is done (See the article on object permanence). More generally, the theory may be very difficult to test empirically because of the difficulty of proving or disproving that a mental model is the explanation for the results of the testing [21]. Neo-Piagetian theories of cognitive development expand Piaget's theory in various ways such as also considering psychometric-like factors such as processing speed and working memory, "hyper cognitive" factors like self-monitoring, more stages, and more consideration on how progress may vary in different domains such as spatial or social [22].

- Parieto-frontal integration theory of intelligence. Based on a review of 37 neuroimaging studies, Jung and Haier proposed that the biological basis of intelligence stems from how well the frontal and parietal regions of the brain communicate and exchange information with each other [23]. Subsequent neuroimaging and lesion studies report general consensus with the theory [24]. A review of the neuroscience and intelligence literature concludes that the parietofrontal integration theory is the best available explanation for human intelligence differences [25].

Investment theory. Based on the Cattell–Horn–Carroll theory, the tests of intelligence most often used in the relevant studies include measures of fluid ability (*gf*) and crystallized ability (*gc*); that differ in their trajectory of development in people [26]. The "investment theory" by Cattell[27] states that the individual differences observed in the procurement of skills and knowledge (*gc*) are partially attributed to the "investment" of *gf*, thus suggesting the involvement of fluid intelligence in every aspect of the learning process [27,28]. The investment theory suggests that personality traits affect "actual" ability, and not scores on an IQ test [29]. Hebb's theory of intelligence suggested a bifurcation as well, Intelligence A (physiological), that could be seen as a semblance of fluid intelligence and Intelligence B (Experiential), similar to crystallized intelligence [30].

Intelligence Compensation Theory (ICT). The intelligence compensation theory [31] states that individuals who are comparatively less intelligent work harder and more methodically, and become more resolute and thorough (more conscientious) in order to achieve goals, to compensate for their "lack of intelligence" whereas more intelligent individuals do not require traits/behaviors associated with the personality factor conscientiousness to progress as they can rely on the strength of their cognitive abilities as opposed to structure or effort [32]. The theory suggests the existence of a causal relationship between intelligence and conscientiousness, such that the development of the personality trait of conscientiousness is influenced by intelligence. This assumption is deemed plausible as it is unlikely that the reverse causal relationship could occur [33]; implying that the negative correlation would be higher between fluid intelligence (*gf*) and conscientiousness. This is justified by the timeline of development of *gf*, *gc*, and personality, as crystallized intelligence would not have developed completely when personality traits develop. Subsequently, during school-going ages, more conscientious children would be expected to gain more crystallized intelligence (Knowledge) through education, as they would be more efficient, thorough, hard-working, and dutiful [34]. This theory has recently been contradicted by evidence that identifies compensatory sample selection which attributes the findings to the bias that comes from selecting samples containing people above a certain threshold of achievement [35].

Bandura's theory of self-efficacy and cognition. The view of cognitive ability has evolved over the years, and it is no longer viewed as a fixed property held by an individual. Instead, the current perspective describes it as a general capacity, comprising not only cognitive, but also motivational, social, and behavioral aspects as well. These facets work together to perform numerous tasks. An essential skill often overlooked is that of managing emotions and aversive experiences that can compromise one's quality of thought and activity. Bandura bridges the link between intelligence and success by crediting individual differences in self-efficacy. Bandura's theory identifies the difference between possessing skills and being able to apply them in challenging situations. The theory suggests that individuals with the same level of knowledge and skill may perform badly, averagely, or excellently based on differences in self-efficacy. A key role of cognition is to allow one to predict events and in turn devise methods to deal with these events effectively. These skills are dependent on processing of unclear and ambiguous stimuli. People must be able to rely on their reserve of knowledge to identify, develop, and execute options. They must be able to apply the learning acquired from previous experiences. Thus, a stable sense of self-efficacy is essential to stay focused on tasks in the face of challenging situations [36]. Bandura's theory of self-efficacy and intelligence suggests that individuals with a relatively low sense of self-efficacy in any field will avoid challenges. This effect is heightened when they perceive the situations as *personal threats*. When failure occurs, they recover from it more slowly than others recover, and credit the failure to an insufficient aptitude. On the other hand, persons with high levels of self-efficacy hold a task diagnostic aim that leads to effective performance [37].

Process, Personality, Intelligence and Knowledge theory (PPIK). Developed by Ackerman, the PPIK (process, personality, intelligence, and knowledge) theory further develops the approach on intelligence as proposed by Cattell, the Investment theory, and Hebb, suggesting a distinction between *intelligence as knowledge* and *intelligence as process* (two concepts that are comparable and related to *gc* and *gf* respectively, but broader and closer to Hebb's notions of "Intelligence A" and "Intelligence B") and integrating these factors with elements such as personality, motivation, and interests [38,39]. Ackerman describes the difficulty of distinguishing process from knowledge, as content cannot be eliminated from any ability test [38-40]. Personality traits are not significantly correlated with the *intelligence as process* aspect except in the context of psychopathology. One exception to this generalization has been the finding of sex differences in cognitive abilities, specifically abilities in mathematical and spatial form [38,41] On the other hand, the *intelligence as knowledge* factor has been associated with personality traits of Openness and Typical Intellectual Engagement [38,42], which also strongly correlate with verbal abilities (Associated with crystallized intelligence) [38].

Latent inhibition. It appears that Latent inhibition, the phenomenon of familiar stimuli having a postponed reaction time when compared with unfamiliar stimuli, has a positive correlation with creativity.

Measuring Human Intelligence

The approach to understanding intelligence with the most supporters and published research over the longest period is based on psychometric testing. It is also by far the most widely used in practical settings [43]. An intelligence quotient (IQ) is a total score derived from a set of standardized tests or subtests designed to assess human intelligence [44]. The abbreviation "IQ"was coined by the psychologist William Stern for the German term *Intelligenzquotient*, his term for a scoring method for intelligence tests at University of Breslau he advocated in a 1912 book [45]. Historically, IQ was a score obtained by dividing a person's mental age score, obtained by administering an intelligence test, by the person's chronological age, both expressed in terms of years and months. The resulting fraction (Quotient) was multiplied by 100 to obtain the IQ score [46]. For modern IQ tests, the raw score is transformed to a normal distribution with mean 100 and standard deviation 15 [47]. This results in approximately two-thirds of the population scoring between IQ 85 and IQ 115 and about 2 percent each above 130 and below 70 [48,49]. Scores from intelligence tests are estimates of intelligence. Unlike, for example, distance and mass, a concrete measure of intelligence cannot be achieved given the abstract nature of the concept of "intelligence" [50]. IQ scores have been shown to be associated with such factors as nutrition [51-53], parental socioeconomic status [54,55], morbidity and mortality [56,57], parental social status [58]. and perinatal environment [59]. While the heritability of IQ has been investigated for nearly a century, there is still debate about the significance of heritability estimates [60,61] and the mechanisms of inheritance [62]. IQ scores are used for educational placement, assessment of intellectual ability, and evaluating job applicants. In research contexts, they have been studied as predictors of job performance [63] and income [64]. They are also used to study distributions of psychometric intelligence in populations and the correlations between it and other variables. Raw scores on IQ tests for many populations have been rising at an average rate that scales to three IQ points per decade since the early 20th century, a phenomenon called the Flynn effect. Investigation of different patterns of increases in subtest scores can also inform current research on human intelligence.

There are a variety of individually administered IQ tests in use in the English-speaking world [65-67]. The most commonly used individual IQ test series is the Wechsler Adult Intelligence Scale (WAIS) for adults and the Wechsler Intelligence Scale for Children (WISC) for school-age test-takers. Other commonly used individual IQ tests (Some of which do not label their standard scores as "IQ" scores) include the current versions of the Stanford-Binet Intelligence Scales, Woodcock-Johnson Tests of Cognitive Abilities, and the Kaufman Assessment Battery for Children, the Cognitive Assessment System, and the Differential Ability Scales.

There are various other IQ tests, including 1. Raven's Progressive Matrices (RPM) 2. Cattell Culture Fair III (CFIT) 3. Reynolds Intellectual Assessment Scales (RIAS)4. Thurston's Primary Mental Abilities [68,69] 5. Kaufman Brief Intelligence Test (KBIT) [70] 6. Multidimensional Aptitude Battery II 7. Das-Naglieri Cognitive Assessment System (CAS) 8. Naglieri Nonverbal Ability Test (NNAT) 9. Wide Range Intelligence Test (WRIT).

IQ scales are ordinally scaled [71-75]. The raw score of the norming sample is usually (Rank order) transformed to a normal distribution with mean 100 and standard deviation 15 [76].

While one standard deviation is 15 points, and two SDs are 30 points, and so on, this does not imply that mental ability is linearly related to IQ, such that IQ 50 would mean half the cognitive ability of IQ 100. In particular, IQ points are not percentage points.

Psychometricians generally regard IQ tests as having high statistical reliability [77,78]. Reliability represents the measurement consistency of a test [79]. A reliable test produces similar scores upon repetition [79]. On aggregate, IQ tests exhibit high reliability, although test-takers may have varying scores when taking the same test on differing occasions, and may have varying scores when taking different IQ tests at the same age. Like all statistical quantities, any particular estimate of IQ has an associated standard error that measures uncertainty about the estimate. For modern tests, the confidence interval can be approximately 10 points and reported standard error of measurement can be as low as about three points [80]. Reported standard error may be an underestimate, as it does not account for all sources of error [81]. Outside influences such as low motivation or high anxiety can occasionally lower a person's IQ test score [82]. For individuals with very low scores, the 95% confidence interval may be greater than 40 points, potentially complicating the accuracy of diagnoses of intellectual disability [83]. By the same token, high IQ scores are also significantly less reliable than those near to the population median [84]. Reports of IQ scores much higher than 160 are considered dubious [84].

Reliability and validity are very different concepts. While reliability reflects reproducibility, validity refers to whether the test measures what it purports to measure [85]. While IQ tests are generally considered to measure some forms of intelligence, they may fail to serve as an accurate measure of broader definitions of human intelligence inclusive of, for example, creativity and social intelligence. For this reason, psychologist Wayne Weiten argues that their construct validity must be carefully qualified, and not be overstated [85]. According to Weiten, "IQ tests are valid measures of the kind of intelligence necessary to do well in academic work. But if the purpose is to assess intelligence in a broader sense, the validity of IQ tests is questionable" [85]. Some scientists have disputed the value of IQ as a measure of intelligence altogether. In *The Mismeasure of Man* (1981, expanded edition 1996), evolutionary biologist Stephen Jay Gould compared IQ testing with the now-discredited practice of determining intelligence via craniometry, arguing that both are based on the fallacy of reification, "our tendency to convert abstract concepts into entities" [86]. Gould's argument sparked a great deal of debate [87,88], and the book is listed as one of *Discover Magazine*'s "25 Greatest Science Books of All Time" [89, 90], Along these same lines, critics such as Keith Stanovich do not dispute the capacity of IQ test scores to predict some kinds of achievement, but argue that basing a concept of intelligence on IQ test scores alone neglects other important aspects of mental ability [77,91] Robert Sternberg, another significant critic of IQ as the main measure of human cognitive abilities, argued that reducing the concept of intelligence to the measure of *g* does not fully account for the different skills and knowledge types that produce success in human society [92]. Despite these objections, clinical psychologists generally regard IQ scores as having sufficient statistical validity for many clinical purposes [93].

Fuzzy Set Model of the Natural Language Expression “Human Intelligence”

The natural language expressions “human intelligence” concern a set of intellectual properties of humans evaluated quantitatively with IQ. However, it is too vague (Fuzzy) to perform computation. Fuzzy modeling provides a means to translate the above natural expression into a computationally usable form. Fuzzy sets are used to describe, imprecisions and uncertainty in case of the fuzzy linguistic expression, in a non-probabilistic (Non-frequentist) framework. In mathematics, fuzzy sets were first introduced by Zadeh [94] in1965, have been applied in various field as decision-making, and control [95,96]. In case of an ordinary set for each object it can be decided whether it belongs or not to the set. A fuzzy set is a collection of objects without well-defined characteristics. In contrast with ordinary sets, a partial membership to a fuzzy set is possible.

The formal definition of a fuzzy set is [97-102]: Let X be an ordinary set (universe) A is called a fuzzy subset of X if A is a set of ordered pairs: $A=\{(x,f_A(x));\ x\in X,\ f_A(x)\in[0,1]\}$The function $f_A: X\rightarrow[0,1]$ is called the membership function of A. $f_A(x)$ is the grade of membership of x in A. The closer $f_A(x)$ is to 1 the more x is considered to belong to A. The closer $f_A(x)$ is to 0 less x is taken to belong to A.

In case of the natural language expression, human intelligence the ordinary set X (Universe) is the set of nonnegative real numbers representing the possible IQ values in case of the abstract concept "Human Intelligence". The function f_A :X→ [0, 1] called the membership function of A, is related to a given IQ test and person. For example Wechsler Adult Intelligence Scale (WAIS), for adults or the Wechsler Intelligence Scale for Children (WISC) for school age test-takers etc. Its values $f_A(x)$ are in the interval [0 ,1]. The fuzzy subset A is the graphic of the function f_A . Because for individuals with very low scores, the 95% confidence interval may be greater than 40 IQ points, potentially complicating the accuracy of diagnoses of intellectual disability. [83] and, high IQ scores are also significantly less reliable than those near to the population median. [84] (reports of IQ scores much higher than 160 are considered dubious. [84]) the universe X will be restraint to the interval [40]

For the membership function, we take a very simple way of defining a triangular fuzzy number A_T with respect to a parameter x is by assessing three numbers [102]:

1. The most credible IQ value x^*-assigned a membership value of 1; that is x^*=100 because two-thirds of the population scoring between IQ values 85 and 115
2. the number x^- which is almost certainly exceeded by the IQ value −assigned a membership value 0; x^- = 40.
3. the number x^+ which is almost certainly not exceeded by the IQ value −assigned a membership value 0; x^+=160.

Let the membership function be defined with 0 outside of the interval (x^- , x^+) of possible values (support) and taken to be piecewise linear in between. The triangular fuzzy subset A_T= $(x^-, x^*, x^+)_T$ represent the fuzzy statement (x is intelligent) and it is the graphic of the membership function

f_{AT} (x)=1/60*(x-40) for 40<x<100 and f_{AT} (x)=1/60*(160-x) for 100<x<160.The graphic is presented on the following figure (Figure 1).

The graphic describs (corresponds) the fuzzy logic statement (x is A) i.e. (x is intelligent) and $f_A(x)$ is the degree of fulfillment DOF of the fuzzy logic statement (x is A) i.e. DOF (x is A) = $f_A(x)$.

Because two-thirds of the population scoring between IQ 85 and IQ 115, and about 2 percent each above 130 and below 70 based on the above fuzzy description we find that for two third of population DOF(x is A_T) is in the interval (0.75, 1). For 2 percent of population DOF(x is A)> 0.7500000000 and for another two percent of the population DOF(x is A_T) < 0.500000000.

The significance of representing the natural language expression "human intelligence" with fuzzy subset A_T allows the use of the computer to find the less intelligent persons, the intelligent persons and the more than intelligent persons in the case of an IQ data set. Thus, if we agree that people with an IQ lower than 70 are less intelligent, those with an IQ between 70 and 121 are intelligent and those with an IQ between 120

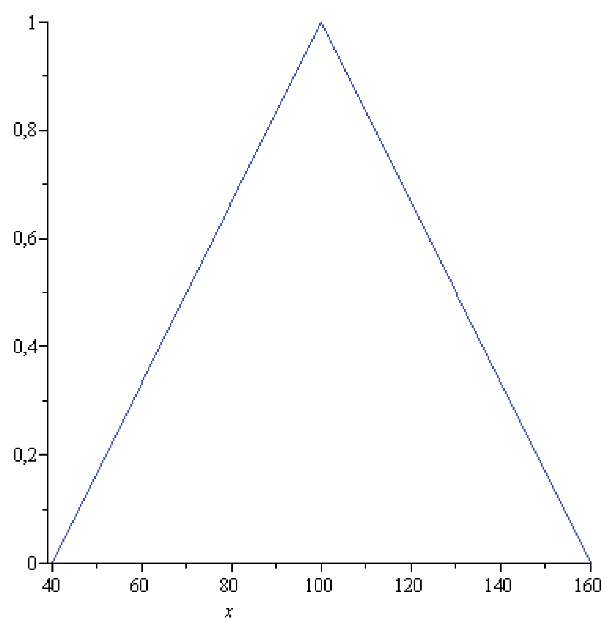

Figure 1 Fuzzy subset A_T representing the fuzzy statement (x is intelligent) in WAIS scale.

and 160 are very intelligent, then in the case of the IQ data set

IQ = [71,74,53,61,41,50,65,72,119,115,119,120,125,124,125,125,129,129,123,49,50,50,47,65,132,135,153,152,42,60,80,81,121,125,132]

is possible to use computer for identify the less intelligent, the intelligent and the more than intelligent persons using only the IQ values and the next results is found:

-less intelligent persons [53,61,41,50,65,49,50,50,47,65,42,60]

-less intelligent persons [71,74,72,119,115,119,120,80,81]

-more than intelligent persons [125,124,125,125,129,129,123,132,135,153,152,121,125,132]

The above identification do not incorporate the 'exigency degree 'of person who make the interpretation. It is possible to incorporate the 'exigency degree 'of person who make the interpretation using for example instead of the IQ values the DOFIQ values.

The first step is the transformation of the IQ data set in the DOFIQ data set using computer and the membership function of the fuzzy subset A_T. The following DOFIQ data set is obtained:

$$DOFIQ := [0.5166666667, 0.5666666667, 0.2166666667, 0.3500000000, 0.01666666667, 0.1666666667, 0.4166666667, 0.5333333333, 0.6833333333, 0.7500000000, 0.6833333333, 0.6666666667, 0.5833333333, 0.6000000000, 0.5833333333, 0.5833333333, 0.5166666667, 0.5166666667, 0.6166666667, 0.1500000000, 0.1666666667, 0.1666666667, 0.1166666667, 0.4166666667, 0.4666666667, 0.4166666667, 0.1166666667, 0.1333333333, 0.03333333333, 0.3333333333, 0.6666666667, 0.6833333333, 0.6500000000, 0.5833333333, 0.4666666667];$$

Because the DOF values of the specified milestones of IQ values 70, 120, 121 are 0.5000000000, 0.6666666667, 0.65 respectively for "less intelligent", "intelligent "and "more than intelligent "with a statistical computer program the following results are found:

"less intelligent" people

$$0.2166666667, 0.3500000000, 0.01666666667, 0.1666666667, 0.4166666667, 0.1500000000, 0.1666666667, 0.1666666667, 0.1166666667, 0.4166666667, 0.4666666667, 0.4166666667, 0.1166666667, 0.1333333333, 0.03333333333, 0.3333333333, 0.4666666667$$

"intelligent" people =

$$[0.5166666667, 0.5666666667, 0.5333333333, 0.5833333333, 0.6000000000, 0.5833333333, 0.5833333333, 0.5166666667, 0.5166666667, 0.6166666667, 0.6500000000, 0.5833333333]$$

"more than intelligent" people=

$$[0.6833333333, 0.7500000000, 0.6833333333, 0.6666666667, 0.6666666667, 0.6833333333]$$

According to this fuzzy representation of the fuzzy verbal expression "intelligent "with a triangular fuzzy set A_T and the use of DOFIQ: the number of the less intelligent people is equal to 17; the number of the intelligent people is equal to 12 and the number of more than intelligent people is equal to 6. On the other hand, according strictly to the IQ value identification: the number of the less intelligent people is equal to 12; the number of the intelligent people is equal to 9 and the number of more than intelligent people is equal to 14. The explanation is that in case of the fuzzy representation of the fuzzy verbal expression "intelligent" the classification is based on the DOFIQ (Not strictly on the IQ value) which incorporate a certain subjectivity of the interpretation. The introduction of DOFIQ in identification make possible to incorporate a type of subjectivity in the interpretation of verbal expression "intelligent" modeled with a triangular fuzzy set A_T.

Linguistic Modifiers

In natural language frequently, a specification of the properties is often done using linguistic modifiers (Hedges) [95]. These modifiers might both increase or decrease the uncertainty. Some of this hedge are: VERY, FAIRLY, MOSTLY, OFTEN, SOMEWHAT, INDEED, ROUGHLY, ALMOST, MORE OR LESS, SORT OFF, PRACTICALLY, NOT, MOST OFF, AT LEAST A FEW. These hedges are applied to fuzzy linguistic expression, resulting in either a more precise or imprecise vague linguistic expression.

The hedge VERY

The membership function is $f_{Very}(x) = f^2_{AT}(x)$ where $f_{AT}(x)$ is the membership function of the fuzzy logic statement (x is intelligent), described by the fuzzy subset A_T (Figure 1). The fuzzy subset A_{Very}, representing the fuzzy logic statement (x is very intelligent) is presented in the next figure (Figure 2).

The significance of representing the natural language expression "very intelligent" with fuzzy subset A_{Very} allows the use of the computer to find: The less than very intelligent, the very intelligent and the more than very intelligent in the case of a given IQ data set. Thus, if we agree that people with an IQ value lower than 70 are "less than very intelligent", those with an IQ value between 70 and 120 are "very intelligent" and those with an IQ value between 120 and 160 are "more than very intelligent", then in the case of the IQ values data set

IQ [71,74,61,41,50,65,72,119,115,119,120,125,124,125,125,129,129,123,49,50,50,47,65,132,135,153,152,42,60,80,81,121,125,132]

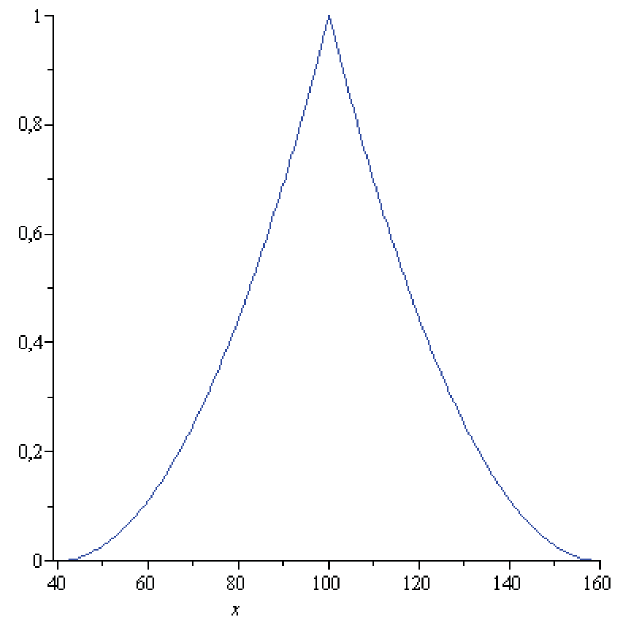

Figure 2 Fuzzy subset A_{Very} representing the fuzzy statement (x is very intelligent) in WAIS scale.

is possible to use computer for find the less than very intelligent, the very intelligent and the more than very intelligent persons.

The first step is the computation of DOFIQ data set using the IQ data set and the membership function of the fuzzy subset A_{Very}. The following DOFIQ data set is obtained:

DOFIQ := [0.2669444444, 0.3211111111, 0.04694444444, 0.1225000000, 0.0002777777778, 0.02777777778, 0.1736111111, 0.2844444444, 0.4669444444, 0.5625000000, 0.4669444444, 0.4444444444, 0.3402777778, 0.3600000000, 0.3402777778, 0.3402777778, 0.2669444444, 0.2669444444, 0.3802777778, 0.02250000000, 0.02777777778, 0.02777777778, 0.01361111111, 0.1736111111, 0.2177777778, 0.1736111111, 0.01361111111, 0.01777777778, 0.001111111111, 0.001111111111, 0.4444444444, 0.4669444444, 0.4225000000, 0.3402777778, 0.2177777778];

Because the DOF values of the specified mile-stones of IQ values 70, 120, are 0.5000000000, 0.6666666667 respectively for "less than very intelligent", "very intelligent "and "more than very intelligent "with a statistical computer program the following results are found:

"less than very intelligent" people=

[0.2669444444, 0.3211111111, 0.04694444444, 0.1225000000, 0.0002777777778, 0.02777777778, 0.1736111111, 0.2844444444, 0.4669444444, 0.4669444444, 0.4444444444, 0.3402777778, 0.3600000000, 0.3402777778, 0.3402777778, 0.2669444444, 0.2669444444, 0.3802777778, 0.02250000000, 0.02777777778, 0.02777777778, 0.01361111111, 0.1736111111, 0.2177777778, 0.1736111111, 0.01361111111, 0.01777777778, 0.001111111111, 0.001111111111, 0.4444444444, 0.4669444444, 0.4225000000, 0.3402777778, 0.2177777778]

"very intelligent people" =[0.5625000000]

"more than very intelligent" people= []

According to the above computation the number of the less than very intelligent people is: equal to 34, the number of the very intelligent people is equal to 1 and the number of more than very intelligent people is equal to 0. On the other hand, according to the representation of "human intelligence": the number of the less than intelligent people is equal to 17; the number of the intelligent people is equal to 11 and the number of more than intelligent people is equal to 7. The mathematical explanation of this significant differences is that in case of the fuzzy representation of the fuzzy verbal expression "very intelligent" the computation is made using the piecewise nonlinear membership function of the A_{Very} fuzzy set and not the piecewise linear membership function of the A_T fuzzy set. The difference reflects the difference between the meaning of the fuzzy linguistic expressions "intelligent" and "very intelligent". The example shows that in classification the difference is significant although the difference between the linguistic expressions intelligent and very intelligent in natural language apparently is not so significant (Is ambiguous).

The hedge MORE OR LESS

The membership function is $f_{More-Less}(x)$=square root $f_{AT}(x)$ where $f_{AT}(x)$ is the membership function of the fuzzy logic statement (x is intelligent), described by the fuzzy subset A_T (Figure 1). The computed fuzzy subset $A_{More-Less}$ representing the fuzzy logic statement (x is more or less intelligent), is presented in the next figure (Figure 3).

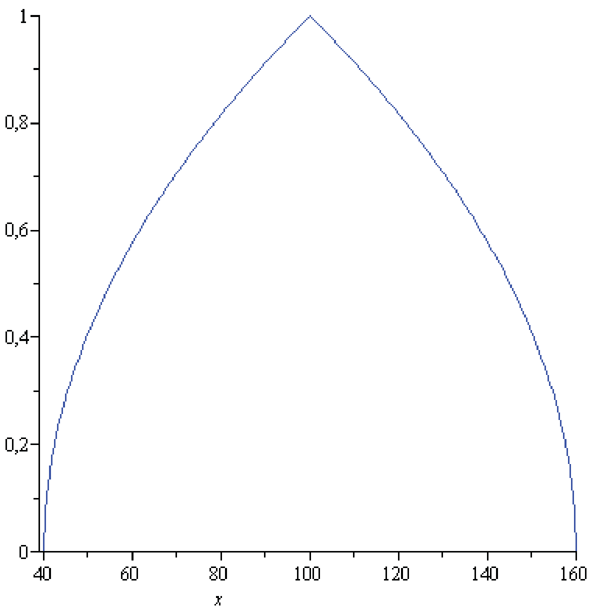

Figure 3 Fuzzy subset $A_{More-Less}$ representing the fuzzy statement (x is more or less intelligent) in WAIS scale.

The significance of representing the natural language expression "more or less intelligence" with fuzzy subset $A_{More-Less}$ allows the use of the computer to identify: the less than more or less intelligent, the more or less intelligent and the more than more or less intelligent in the case of a given IQ data set. Thus, if we agree that people with an IQ lower than 70 are "less than more or less intelligent", those with an IQ between 70 and 102 are "more or less intelligent" and those with an IQ between 102 and 160 are "more than more or less intelligent", then in the case of IQ values data set

1Q =[71,74,53,61,41,50,65,72,119,115,119,120,125,124,125,125,129,129,123,49,50,50,47,65,132,135,152,42,60,80,81,121,125,132];

is possible to use computer for identify: the less than more or less intelligent; the more or less intelligent and the more than more or less intelligent.

The first step is the computation of DOFIQ data set using the membership function of the fuzzy subset $A_{More-Less.}$ The following DOFIQ data set is obtained:

$$DOFIQ := [0.7187952883, 0.7527726526, 0.4654746680, 0.5916079783, 0.1290994449, 0.4082482906, 0.6454972245, 0.7302967432, 0.8266397846, 0.8660254040, 0.8266397846, 0.8164965809, 0.7637626160, 0.7745966692, 0.7637626160, 0.7637626160, 0.7187952883, 0.7187952883, 0.7852812659, 0.3872983346, 0.4082482906, 0.4082482906, 0.3415650256, 0.6454972245, 0.6831300514, 0.6454972245, 0.3415650256, 0.3651483717, 0.1825741858, 0.1825741858, 0.8164965809, 0.8266397846, 0.8062257748, 0.7637626160, 0.68313005 1];$$

Because the DOF values of the specified milestones of IQ values 70, 120, are 0.5000000000, 0.6666666667 respectively for "less than more or less intelligent", "more or less intelligent "and "more than more or less intelligent "the following results are found:

"less than more or less intelligent" people=

$$[0.4654746680, 0.1290994449, 0.4082482906, 0.3872983346, 0.4082482906, 0.4082482906, 0.3415650256, 0.3415650256, 0.3651483717, 0.1825741858, 0.1825741858]$$

"more or less intelligent" people=

0.7187952883, 0.7527726526, 0.5916079783, 0.6454972245,
0.7302967432, 0.8266397846, 0.8660254040, 0.8266397846,
0.8164965809, 0.7637626160, 0.7745966692, 0.7637626160,
0.7637626160, 0.7187952883, 0.7187952883, 0.7852812659,
0.6454972245, 0.6831300514, 0.6454972245, 0.8164965809,
0.8266397846, 0.8062257748, 0.7637626160, 0.683130051

"more than more or less intelligent" people=

0.7187952883, 0.7527726526, 0.7302967432, 0.8266397846,
0.8660254040, 0.8266397846, 0.8164965809, 0.7637626160,
0.7745966692, 0.7637626160, 0.7637626160, 0.7187952883,
0.7187952883, 0.7852812659, 0.6831300514, 0.8164965809,
0.8266397846, 0.8062257748, 0.7637626160, 0.683130051

The number of the less than more or less intelligent people is equal to 11; the number of the more or less intelligent people is equal to 24; and the number of more than more or less intelligent people is equal to 20. On the other hand, according to the results concerning classification obtained using A_T fuzzy set: the number of the less than intelligent people is equal to 17; the number of the intelligent people is equal to 11 and the number of more than intelligent people is equal to 7. The mathematical explanation of differences is that in case of the fuzzy representation of the fuzzy verbal expression "more or less intelligent" the classification is based on the piecewise nonlinear membership function of $A_{More\text{-}Less}$ fuzzy set and not on the piecewise linear membership function of A_T fuzzy set. The differences reflect the difference between the meaning of the fuzzy linguistic expressions "intelligent" and "more or less intelligent". The above example shows that in classification the difference is significant although the difference between the linguistic expressions intelligent and more or less intelligent in natural language apparently is not so significant.

The hedge INDEED

The membership function is $f_{Indeed}(x) = 2*f^2_{AT}(x)$ for $f_{AT}(x) < 0.5$ and $f_{Indeed}(x) = 1-2*(1-f_{AT}(x))^2$ for $f_{AT}(x) > 0.5$. Where $f_{AT}(x)$ represent the membership function in case of the fuzzy logic statement (x is intelligent) described by the fuzzy subset A_T (Figure 1). The fuzzy subset A_{Indeed}, representing the fuzzy logic statement (x is indeed intelligent), is presented in the next figure (Figure 4).

The significance of representing the natural language expression "indeed intelligent" with fuzzy subset A_{Indeed} allows the use of the computer to identify: the "less than indeed intelligent", the "indeed intelligent" and , the "more than indeed intelligent" in the case of a given IQ data set. Thus, if we agree that people with an IQ lower than 80 are "less than indeed intelligent", those with an IQ between 70 and 120 are "indeed intelligent" and those with an IQ between 120 and 160 are "more than indeed intelligent", then in the case of the IQ data set

$IQ := [71, 74, 53, 61, 41, 50, 65, 72, 119, 115, 119, 120, 125, 124, 125, 125, 129, 129, 123, 49, 50, 50, 47, 65, 132, 135, 153, 152, 42, 60, 80, 81, 121, 125, 132];$

is possible to use computer for identify: the less than indeed intelligent; the indeed intelligent and the more than indeed intelligent persons.

The first step is the computation of DOFIQ data set using the membership function of the

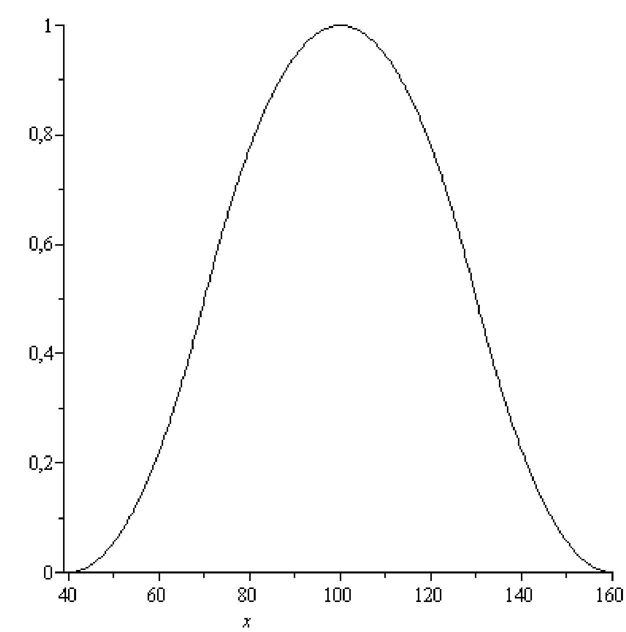

Figure 4 Fuzzy subset A_{Indeed} representing the fuzzy statement (x is indeed intelligent) in WAIS scale.

fuzzy subset $A_{Indeed.}$ The following DOFIQ data set is obtained:

$DOFIQ$:= [0.5327777778, 0.6244444444, 0.09388888889, 0.2450000000, 0.0005555555556, 0.05555555556, 0.3472222222, 0.5644444444, 0.7994444444, 0.5625000000, 0.4669444444, 0.7777777778, 0.6527777778, 0.6800000000, 0.6527777778, 0.3402777778, 0.5327777778, 0.5327777778, 0.3802777778, 0.04500000000, 0.05555555556, 0.05555555556, 0.02722222222, 0.3472222222, 0.4311111111, 0.3194444444, 0.02722222222, 0.03555555556, 0.002222222222, 0.002222222222, 0.7777777778, 0.7994444444, 0.7550000000, 0.6527777778, 0.4355555556];

Because the DOF values of the specified milestones of IQ values 70, 120, are 0.5000000000, 0.6666666667 respectively for "less than more or less intelligent", "more or less intelligent "and "more than more or less intelligent "the following results are found:

"less than indeed intelligent" people:

0.09388888889, 0.2450000000, 0.0005555555556, 0.05555555556, 0.3472222222, 0.4669444444, 0.3402777778, 0.3802777778, 0.04500000000, 0.05555555556, 0.05555555556, 0.02722222222, 0.3472222222, 0.4311111111, 0.3194444444, 0.02722222222, 0.03555555556, 0.002222222222, 0.002222222222, 0.4355555556

"indeed intelligent" people:

[0.5327777778, 0.6244444444, 0.5644444444, 0.7994444444, 0.5625000000, 0.7777777778, 0.6527777778, 0.6800000000, 0.6527777778, 0.5327777778, 0.5327777778, 0.7777777778, 0.7994444444, 0.7550000000, 0.6527777778]

"more than indeed intelligent" people:

[0.7994444444, 0.7777777778, 0.6800000000, 0.7777777778, 0.7994444444, 0.7550000000]

The number of less than indeed intelligent people is equal to 20; the number of indeed intelligent people is equal to 15, the number of more than indeed intelligent people is 6. On the other hand, according to the results, concerning classification obtained using A_T fuzzy set the number of the less than intelligent people is equal to 17; the number of the intelligent people is equal to 11 and the number of more than intelligent people is equal to 7. The mathematical explanation of differences is that in case of the fuzzy representation of the fuzzy verbal expression "indeed intelligent" the classification is based on the piecewise nonlinear membership function of A_{Indeed} fuzzy set and not on the piecewise linear membership function of A_T fuzzy set. The difference in classification reflect the difference between the meaning of the fuzzy linguistic expressions "intelligent" and "indeed intelligent". The above example shows that in classification the difference is significant although the difference between the linguistic expressions intelligent and indeed intelligent in natural language apparently is not so significant.

A rough representation of the difference between the fuzzy subsets A_T, A_{Very}, $A_{More\text{-}Less}$ and A_{Indeed} corresponding to the fuzzy logic statements (x is intelligent), (x is very intelligent), (x is more or less intelligent) ,and (x is indeed intelligent) respectively can be seen in the next figure where A_T , A_{Very}, $A_{More\text{-}Less}$ and A_{Indeed} are represented with colors red, blue , green and black respectively (Figure 5).

It can be seen that: in case of the fuzzy logic statement (x is very intelligent) the membership value of all the uncertain elements is less

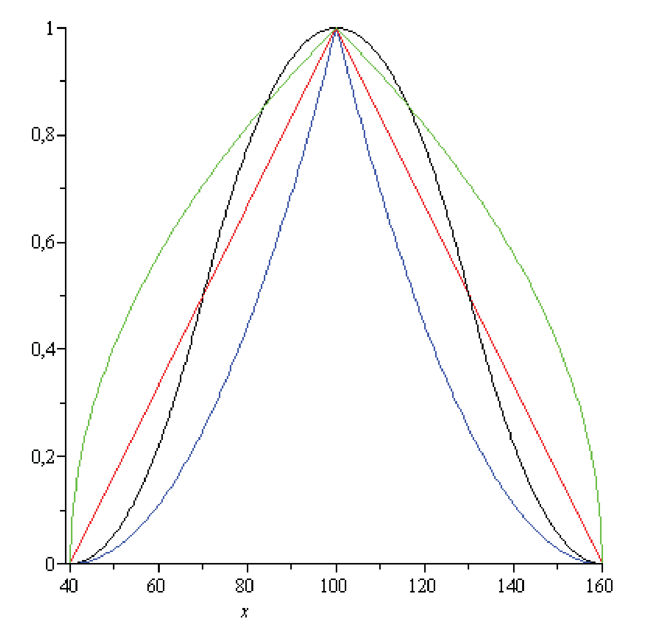

Figure 5 Fuzzy sets corresponding to the fuzzy logic statements: (x is intelligent) color red, (x is very intelligent) color blue, (x is more or less intelligent) color green, (x is indeed intelligent) color black.

than in case of the fuzzy logic statement (x is intelligent); in case of the fuzzy logic statement (x is more or less intelligent) the membership of all the uncertain elements is more than in the case of the fuzzy logic statement (x is intelligent); in case of the fuzzy logic statement (x is indeed intelligent) the membership value of uncertain elements for which DOF(x is A_T) < 0.500000000 implies DOF(x is A_{Indeed}) < DOF(x is A_T) and for those x for which DOF(x is A_T) > 0.500000000 implies DOF(x is A_{Indeed}) > DOF(x is A_T).Mathematically these differences are generated by the choice of interpolation of the values 0 at $x^- = 40$,1 at $x^* = 100$ and 0 at $x^+ = 160$. In case of A_T the interpolation is piecewise linear; in case of A_{Very}, $A_{More - Less}$ and A_{Indeed} is nonlinear. If A_T describes the understanding of general intelligence then: A_{Very} describes a more exigent understanding of the general intelligence; $A_{More-Less}$ describes a less exigent understanding of the general intelligence; A_{Indeed} represent a more exigent understanding of the general intelligence for the IQ values in the intervals [40,70] and [130,160] ; and A_{Indeed} represent a less exigent understanding of the general intelligence for the IQ values in the interval [70,130]. Note that the exigency is reflected in DOFIQ and not in IQ.

Human Intelligence as Linguistic Variable

According to [97] taking: the set T of the above presented four natural language terms "intelligent", "very intelligent", "more or less intelligent", "indeed intelligent"; the universe X equal to the interval [40,160] i.e. X = [40,160]; G the context free grammar {Intelligent, very intelligent, more or less intelligent, indeed intelligent} used to generate the elements of T; and the map M from T to the set of the corresponding fuzzy subsets F={ A_T , A_{Very}, $A_{More-Less}$ and A_{Indeed} } which associate to the elements of T the corresponding fuzzy subset from F, a linguistic variable is obtained what we will call "THE LINGUISTIC VARIABLE HUMAN INTELLIGENCE "This linguistic variable can be extended defining for example membership functions for the other linguistic modifiers.

In general, the formal definition of a linguistic variable Y is: a 4-tuple (T, X, G, M) where T is a set of natural language terms from which t can take on its values, X is a universe, a common set on which the fuzzy sets corresponding to the linguistic variable are defined, G is a (context free) grammar used to generate the elements of T, and M is a mapping from T to the fuzzy subsets of X. M: T→F [97]. Linguistic variables make the natural language computation possible.

Sometimes there is no set X that can be naturally associated to the linguistic expression. That is because there is no measure for them. Consider for example the linguistic expression; good, pain, happy, joy, excellent, acceptable, etc.

Fuzzy Logic and Fuzzy Logic Operators [102]

In classic logic, a statement is true or false. For this reason, in Boolean mathematical logic two values 0 (False) and 1 (True) are assigned to any statement. In the following table, the true values are given in case of the application of different logical operators (Table 1).

where XOR stands for "either..., or,...."

In fuzzy logic no explicit functional form is assumed, binary logic is replaced by fuzzy logic where a statement and its opposite may both be "true" to a certain degree. For example, "severe" and "moderate" pathology may be both be "true "for a given patient. For

fuzzy statements A, B the "true value" can vary between 0 and 1. The Boolean table has to

Table 1: Logical operations truth table.

A	B	not	A A (AND)	B A (OR)	B A (XOR)	B A (implyB)
1	1	0	1	1	0	1
1	0	0	0	1	1	0
0	1	1	0	1	1	1
0	0	1	0	0	0	1

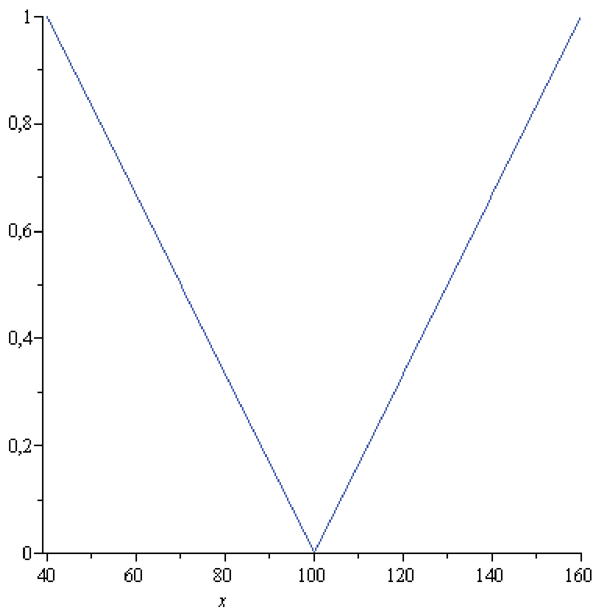

Figure 6 Fuzzy subset C= A_T^C representing the fuzzy logic statement (x is not intelligent).

be extended to cope with such situations in a plausible manner.

Fuzzy logic operator NOT

In case of the fuzzy statement (x is A) the fuzzy logic operator NOT transform this statement in the fuzzy statement (x is not A). The new fuzzy statement usually is denoted by NOT (x is A). The fuzzy subset which represent the fuzzy statement (x is not A) membership function is equal to the function 1-f_A(x). This fuzzy subset usually is denoted by C=A^c and is called complement of A. It can be seen that DOF (NOT (x is A)) = f_C(x).

In the case when A is A_T (the intelligence of a person in WAIS scale) DOF (NOT (x is A_T)) = 1-1/60*(x-40) for 40 < x < 100 and 1-1/60*(160-x) for 100 < x < 160. The complement C = A_T^C is the fuzzy set which represent the fuzzy statement Not (x is A_T) and is presented in the next figure (Figure 6).

When A is A_{Very}, $A_{More-Less}$, A_{Indeed} the fuzzy subsets $C_{Avery}{}^C$, $C_{Amore-Less}{}^C$, $C_{AIndeed}{}^C$ are represented in the following figures (Figures 7-9):

Fuzzy logic operator AND

According to [102] in fuzzy logic two type of AND logic operators are used: the so called "min fuzzy logic operator AND" and the so called "product fuzzy logic operator AND".

Min fuzzy logic operator AND

In case of the fuzzy statements (x is A_1), (x is A_2) the "min fuzzy logic operator AND" transform

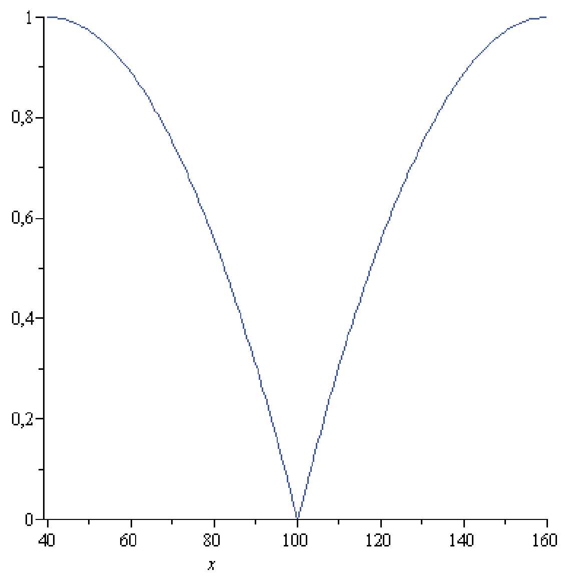

Figure 7 Subset $C_{Avery}{}^C$.

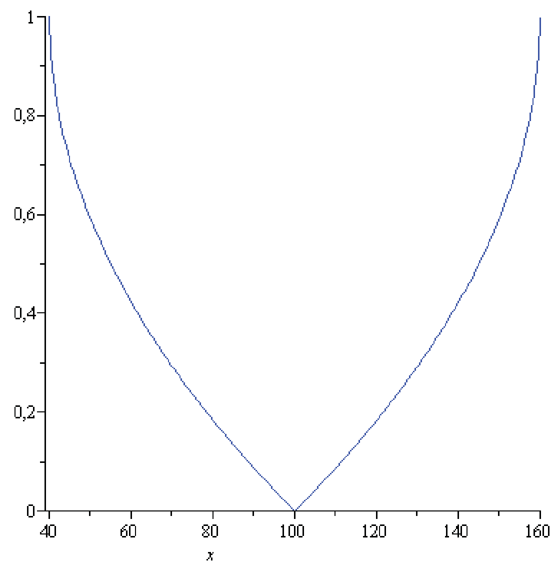

Figure 8 Subset $C_{Amore-Less}{}^C$.

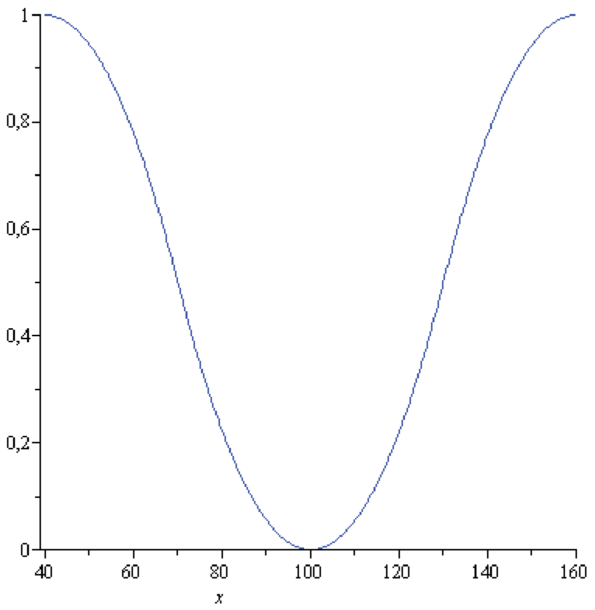

Figure 9 Subset $C_{AIndeed}{}^{C}$.

this statements in the fuzzy statement (x is A_1) AND(x is A_2)denoted usually by" min(x is A_1) AND(x is A_2) " The fuzzy subset C which represent the fuzzy statement " min(x is A_1)AND(x is A_2) " membership function is $f_{Cmin}(x) = \min\{f_{A1}(x), f_{A2}(x)\}$. This fuzzy subset usually is denoted by $C_{min} = \min(A_1 \cap A_2)$ and is called the "min fuzzy intersection" of fuzzy subsets A_1 and A_2 . It can be seen that the membership function of the so called "min fuzzy intersection" $C_{min} = A_1 \cap A_2$ is $f_{Cmin}(x) = \min\{f_{A1}(x), f_{A2}(x)\}$. According to this definition is easy to see that in case of the "min fuzzy intersection" the following equalities hold $\min(A_T \cap A_{Very}) = A_{Very}$, $\min(A_T \cap A_{Mor\text{-}Less}) = A_T$,$\min(A_{Very} \cap A_{Mor\text{-}Les}) = A_{Very}$, $\min(A_{Very} \cap A_{Indeed}) = A_{Very}$ (Figure 1). However, in general the "min fuzzy intersection" C_{min} require the computation of the membership function f_{Cmin}. This task can be accomplished using one of the existing computer programs.

For $C_{min} = \min(A_T \cap C_{AT})$ the following membership function is found $f_{Cmin}(x) = (x-40)/60$ for $40 < x < 70$ and $(160-x)/60$ for $130 < x < 160$ and $f_{Cmin}(x) = 1-(x-40)/60$ for $70 < x < 100$ and $1-(160-x)/60$ for $100 < x < 130$.The fuzzy subset C_{min} is presented in the next figure (Figure 10).

We emphasize that for the IQ values 70 and 130 the DOF of the statements min (x is intelligent) AND (x is not intelligent) is equal to 0.5. This situation is similar with that mentioned above: "severe" and "moderate" pathology may be both be "true "for a given patient.

For $C_{min} = \min(A_T \cap A_{Indeed})$ the following

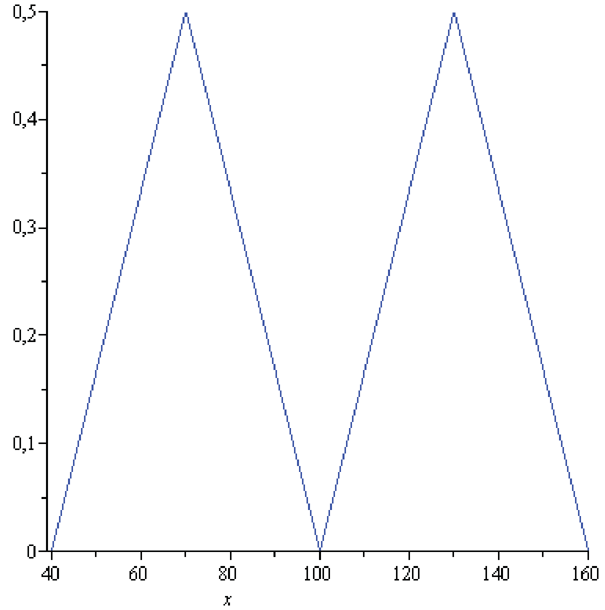

Figure 10 Fuzzy subset $C_{min} = A_T \cap C_{AT}$.

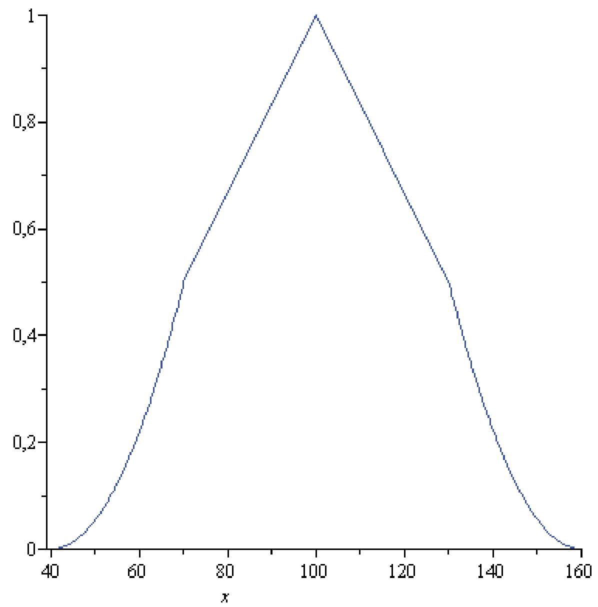

Figure 11 "Min fuzzy intersection" fuzzy subset $C_{min} = \min(AT \cap {}_{AIndeed})$.

membership function is found: $f_{Cmin}(x) = 2*[(x-40)/60]^2$ for $40 < x < 70$ and $f_{Cmin}(x) = 2*[(160-x)/60]^2$ for $130 < x < 160$ and $f_{Cmin}(x) = 1-2*[(100-x)/60]^2$ for $70 < x < 100$ and $f_{Cmin}(x) = 1-2*[(x-100)/60]^2$ for $100 < x < 130$ (Figure 11).

The significance of representing the fuzzy statement "min(x is A_T)AND(x is A_{Indeed})" with fuzzy subset $C_{min} = \min(A_T \cap A_{Indeed})$ allows the use of the computer to find persons having the DOF in a given interval in the framework. min "(x is intelligent) AND (x is indeed intelligent)" in case of an IQ data set. Thus, if we agree that the IQ data set is:

IQ := [71, 74, 53, 61, 41, 50, 65, 72, 119, 115, 119, 120, 125, 124, 125, 125, 129, 129, 123, 49, 50, 50, 47, 65, 132, 135, 153, 152, 42, 60, 80, 81, 121, 125, 132];

then using the membership function of $C_{min} = \min(A_T \cap A_{Indeed})$ we find the following set of DOFIQ:

DOFIQ := [0.5166666667 0.5666666667 0.04694444444 0.1225000000 0.0002777777778 0.02777777778 0.1736111111 0.5333333333 0.6833333333 0.7500000000 0.6833333333 0.6666666667 0.5833333333 0.6000000000 0.5833333333 0.5833333333 0.5166666667 0.5166666667 0.6166666667 0.02250000000 0.02777777778 0.02777777778 0.01361111111 0.1736111111 0.2177777778 0.1736111111 0.01361111111 0.1333333333 0.001111111111 0.001111111111 0.6666666667 0.6833333333 0.6500000000 0.5833333333 0.2177777778]

The above DOFIQ make possible to use computer program for find persons having DOFIQ in a prior given region.

There are 29 persons having DOFIQ in the interval (0 , 0.6666666667) .Namely:

0.5166666667 0.5666666667 0.04694444444 0.1225000000 0.0002777777778 0.02777777778 0.1736111111 0.5333333333 0.5833333333 0.6000000000 0.5833333333 0.5833333333 0.5166666667 0.5166666667 0.6166666667 0.02250000000 0.02777777778 0.02777777778 0.01361111111 0.1736111111 0.2177777778 0.1736111111 0.01361111111 0.1333333333 0.001111111111 0.001111111111 0.6500000000 0.5833333333 0.2177777778

- There are 18 persons having DOFIQ in the interval (0.5 , 1) :Namely:

0.5166666667 0.5666666667 0.5333333333 0.6833333333 0.7500000000 0.6833333333 0.6666666667 0.5833333333 0.6000000000 0.5833333333 0.5833333333 0.5166666667 0.5166666667 0.6166666667 0.6666666667 0.6833333333 0.6500000000 0.5833333333

- There are 4 persons having DOFIQ in the interval (0.6666666667 , 1). Namely:

[0.6833333333 0.7500000000 0.6833333333 0.6833333333]

Product fuzzy logic operator AND

The "product fuzzy logic operator AND" in case of the fuzzy statements (x is A_1), (x is A_2) transform this statements in the fuzzy statement (x is A_1) AND (x is A_2) denoted usually by" prod (x is A_1) AND (x is A_2) "The fuzzy subset D_{prod}, which represent the fuzzy statement" prod (x is A_1) AND (x is A_2) "membership function is $f_{Dprod}(x) = f_{A1}(x)* f_{A2}(x)$. This fuzzy subset usually is denoted by $D_{prod} = prod(A_1 \cap A_2)$ and is called the "prod fuzzy intersection" of fuzzy subsets A_1 and A_2 .

The DOF of x from D_{prod} is given by DOF (prod (x is A_1) AND (x is A_2)) = $f_{A1}(x)* f_{A2}(x)$. If A_1 is the fuzzy subset A_T of intelligence in WAIS scale and A_2 is the fuzzy subset A_{Very} of very intelligence, then their "prod fuzzy intersection" is presented in the next figure (Figure 12).

The significance of representing the fuzzy statement "prod(x is A_T)AND(x is A_{Very})" with

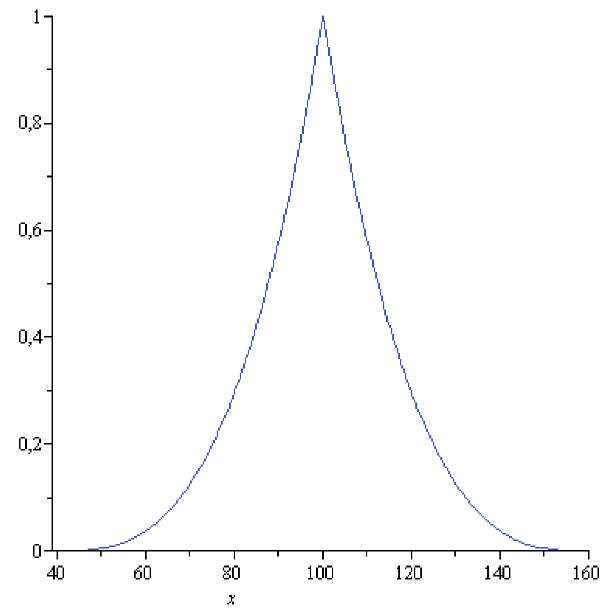

Figure 12 "Prod. fuzzy intersection" Dprod = $A_T \cap A_{Very}$.

fuzzy subset $D_{prod} = A_T \cap A_{Very}$ allows the use of the computer to find the DOF of the fuzzy statement prod "(x is intelligent) AND (x is very intelligent)" of persons in case an IQ data set. Thus, if we agree that the IQ data set is:

IQ := [71, 74, 53, 61, 41, 50, 65, 72, 119, 115, 119, 120, 125, 124, 125, 125, 129, 129, 123, 49, 50, 50, 47, 65, 132, 135, 153, 152, 42, 60, 80, 81, 121, 125, 132];

then using the membership function of $D_{prod} = A_T \cap A_{Very}$ we find the following set of DOFIQ :

$DOFIQ$:= [0.1379212963, 0.1819629630, 0.01017129630, 0.04287500000, 0.000004629629630, 0.004629629630, 0.07233796296, 0.1517037037, 0.3190787037, 0.4218750000, 0.3190787037, 0.2962962963, 0.1984953704, 0.2160000000, 0.1984953704, 0.1984953704, 0.1379212963, 0.1379212963, 0.2345046296, 0.003375000000, 0.004629629630, 0.004629629630, 0.001587962963, 0.07233796296, 0.1016296296, 0.07233796296, 0.001587962963, 0.002370370370, 0.00003703703704, 0.00003703703704, 0.2962962963, 0.3190787037, 0.2746250000, 0.1984953704, 0.1016296296];

The above DOFIQ make possible to use computer program for find persons having DOFIQ in a prior given region.

- There are 35 persons (all) having DOFIQ in the interval (0,0.5) .Namely:

0.1379212963, 0.1819629630, 0.01017129630, 0.04287500000, 0.000004629629630, 0.004629629630, 0.07233796296, 0.1517037037, 0.3190787037, 0.4218750000, 0.3190787037, 0.2962962963, 0.1984953704, 0.2160000000, 0.1984953704, 0.1984953704, 0.1379212963, 0.1379212963, 0.2345046296, 0.003375000000, 0.004629629630, 0.004629629630, 0.001587962963, 0.07233796296, 0.1016296296, 0.07233796296, 0.001587962963, 0.002370370370, 0.00003703703704, 0.00003703703704, 0.2962962963, 0.3190787037, 0.2746250000, 0.1984953704, 0.1016296296

- There are 0 persons having DOFIQ in the interval (0.5, 1).

[]

- There are 26 persons having DOFIQ in the interval (0, 0.2). Namely:

0.1379212963, 0.1819629630, 0.01017129630, 0.04287500000, 0.000004629629630, 0.004629629630, 0.07233796296, 0.1517037037, 0.1984953704, 0.1984953704, 0.1984953704, 0.1379212963, 0.1379212963, 0.003375000000, 0.004629629630, 0.004629629630, 0.001587962963, 0.07233796296, 0.1016296296, 0.07233796296, 0.001587962963, 0.002370370370, 0.00003703703704, 0.00003703703704, 0.1984953704, 0.1016296296

Comparing the identification results obtained using the "min fuzzy logic operator AND" with that obtained using the "prod. fuzzy logic operator AND" it is easy to see that the difference is significant. These differences follows from the two different meaning of the fuzzy logic operator AND.

Fuzzy logic operator OR

According to [102] in fuzzy logic, two type of logic operator OR are used: the so called "max fuzzy logic operator OR" and a so called "product fuzzy logic operator OR"

Max fuzzy logic operator OR

In case of the fuzzy statements (x is A_1), (x is A_2) the "max fuzzy logic operator OR" transform this statements in the fuzzy statement (x is A_1) OR (x is A_2) denoted usually with" max (x is A_1) AND (x is A_2) "The fuzzy subset C_{max} which represent the fuzzy statement" max (x is A_1) OR (x is A_2) "membership function is $f_{Cmax}(x) = \max\{f_{A1}(x), f_{A2}(x)\}$. This fuzzy subset usually is denoted by $C_{max} = \max(A_1 \cup A_2)$ and is called the "max fuzzy union" of fuzzy subsets A_1 and A_2 . The DOF of x from C_{max} is given by DOF (max (x is A_1) OR (x is A_2)) = $\max\{f_{A1}(x), f_{A2}(x)\}$. If A_1 is the fuzzy subset A_T of intelligence in WAIS scale and A_2 is the fuzzy subset A_{Indeed} of indeed intelligent then their "max fuzzy union" is presented in the next figure (Figure 13).

The significance of representing the fuzzy statement "max(x is A_1)OR(x is A_2)" with fuzzy subset $C_{max} = A_T \cup A_{Indeed}$ allows the use of the computer to find the DOF of the fuzzy statement max "(x is intelligent) OR (x is indeed intelligent)" of persons in case of an IQ data set. Thus, if we agree that the IQ data set is:

IQ := [71, 74, 53, 61, 41, 50, 65, 72, 119, 115, 119, 120, 125, 124, 125, 125, 129, 129, 123, 49, 50, 50, 47, 65, 132, 135, 153, 152, 42, 60, 80, 81, 121, 125, 132];

then using the membership function of $C_{max} = A_T \cup A_{Indeed}$ the following set of DOFIQ is found:

$DOFIQ$:= [0.5327777778, 0.6244444444, 0.09388888889, 0.2450000000, 0.0005555555556, 0.05555555556, 0.3472222222, 0.5644444444, 0.7994444444, 0.8750000000, 0.7994444444, 0.7777777778, 0.6527777778, 0.6800000000, 0.6527777778, 0.6527777778, 0.5327777778, 0.5327777778, 0.7061111111, 0.04500000000, 0.05555555556, 0.05555555556, 0.02722222222, 0.1736111111, 0.4355555556, 0.3472222222, 0.02722222222, 0.03555555556, 0.002222222222, 0.002222222222, 0.7777777778, 0.6833333333, 0.7550000000, 0.6527777778, 0.4355555556];

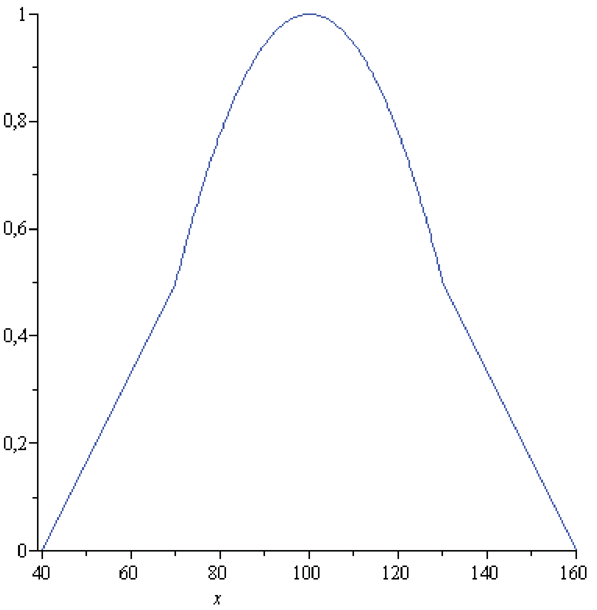

Figure 13 ”max. fuzzy union” Cmax = $A_T \cap A_{Indeed}$.

The above DOFIQ make possible to use computer program for find persons having DOFIQ in a prior given region.

There are 26 persons having DOFIQ in the interval (0,0.6666666667). Namely:

0.5327777778, 0.6244444444, 0.09388888889, 0.2450000000, 0.0005555555556, 0.05555555556, 0.3472222222, 0.5644444444, 0.6527777778, 0.6527777778, 0.6527777778, 0.5327777778, 0.5327777778, 0.04500000000, 0.05555555556, 0.05555555556, 0.02722222222, 0.1736111111, 0.4355555556, 0.3472222222, 0.02722222222, 0.03555555556, 0.002222222222, 0.002222222222, 0.6527777778, 0.4355555556

There are 18 persons having DOFIQ in the interval (0.5, 1). Namely:

[0.5327777778, 0.6244444444, 0.5644444444, 0.7994444444, 0.8750000000, 0.7994444444, 0.7777777778, 0.6527777778, 0.6800000000, 0.6527777778, 0.6527777778, 0.5327777778, 0.5327777778, 0.7061111111, 0.7777777778, 0.6833333333, 0.7550000000, 0.6527777778]

There are 9 persons having DOFIQ in the interval (0.6666666667,1). Namely:

[0.7994444444, 0.8750000000, 0.7994444444, 0.7777777778, 0.6800000000, 0.7061111111, 0.7777777778, 0.6833333333, 0.7550000000]

Product fuzzy logic operator OR

In case of the fuzzy statements (x is A_1), (x is A_2) the “product fuzzy logic operator OR” transform this fuzzy statements in the fuzzy statement (x is A_1) OR (x is A_2) denoted usually with” prod(x is A_1)OR(x is A_2) “ The fuzzy subset D_{prod} which represent the fuzzy statement ” prod(x is A_1)OR(x is A_2) “ membership function is $f_{Dprod}(x)$ =. $f_{A1}(x)+ f_{A2}(x)-f_{A1}(x)*f_{A2}(x)$. This fuzzy subset usually is denoted by D_{prod} = prod($A_1 \cup A_2$) and is called the “prod fuzzy union” of fuzzy subsets A_1 and A_2 . The DOF of x from D_{prod} is given by DOF (prod (x is A_1) OR (x is A_2)) = $f_{A1}(x)+ f_{A2}(x)-f_{A1}(x)*f_{A2}(x)$. If A_1 is the fuzzy subset A_T of intelligence in WAIS scale and A_2 is the fuzzy subset A_{Indeed} of indeed intelligent then their “prod fuzzy union” is presented in the next figure (Figure 14).

The significance of representing the fuzzy statement “prod(x is A_1)OR(x is A_2)” with fuzzy subset $D_{prod} = A_T \cup A_{Indeed}$ allows the use of the computer to find the DOF of the fuzzy statement prod “(x is intelligent) OR (x is indeed intelligent)” of persons in case of an IQ data set. Thus, if the IQ data set is:

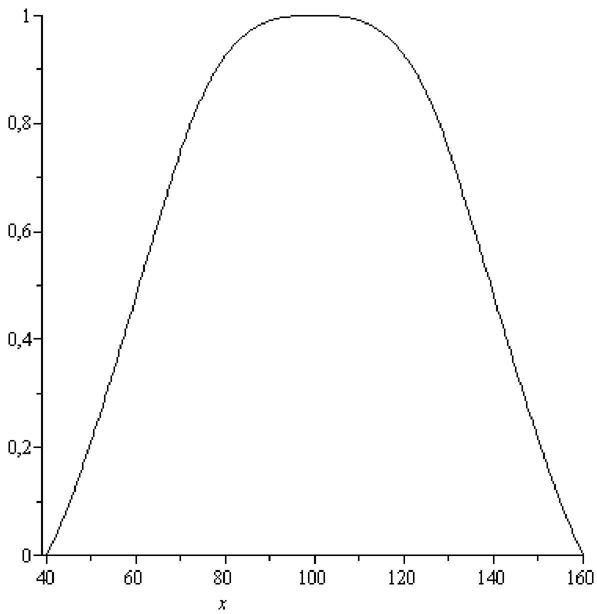

Figure 14 “prod. fuzzy union” Dprod = $A_T \cap A_{Indeed}$.

$IQ :=$ [71, 74, 53, 61, 41, 50, 65, 72, 119, 115, 119, 120, 125, 124, 125, 125, 129, 129, 123, 49, 50, 50, 47, 65, 132, 135, 153, 152, 42, 60, 80, 81, 121, 125, 132];

then using the membership function of $D_{prod} = A_T$

[0.4989074074, 0.4834074074, 0.2698703704, 0.4235000000, 0.01720370370, 0.2037037037, 0.4745370370, 0.4957037037, 0.3902037037, 0.3125000000, 0.3902037037, 0.4074074074, 0.4745370370, 0.4640000000, 0.4745370370, 0.4745370370, 0.4989074074, 0.4989074074, 0.4519074074, 0.1815000000, 0.2037037037, 0.2037037037, 0.1375370370, 0.4745370370, 0.4957037037, 0.4745370370, 0.1375370370, 0.1594074074, 0.03540740741, 0.03540740741, 0.4074074074, 0.3902037037, 0.4235000000, 0.4745370370, 0.4957037037];

The above DOFIQ make possible to use computer program for find persons having DOFIQ in a prior given region.

There are 15 persons having DOFIQ in the interval (0, 0.6666666667). Namely:

[0.2902129630, 0.5092500000, 0.01721296296, 0.2129629630, 0.6192129630, 0.1882500000, 0.2129629630, 0.2129629630, 0.1407129630, 0.6192129630, 0.6192129630, 0.1407129630, 0.1641481481, 0.03548148148, 0.03548148148]

– There are 24 persons having DOFIQ in the interval (0.5, 1). Namely:

0.7741759259, 0.8372592593, 0.5092500000, 0.6192129630, 0.7967407407, 0.9364907407, 0.9687500000, 0.9364907407, 0.9259259259, 0.8553240741, 0.8720000000, 0.8553240741, 0.8553240741, 0.7741759259, 0.7741759259, 0.8873425926, 0.6192129630, 0.6989629630, 0.6192129630, 0.9259259259, 0.9364907407, 0.9142500000, 0.8553240741, 0.6989629630

– There are 20 persons having DOFIQ in the interval (0.6666666667, 1). Namely:

0.7741759259, 0.8372592593, 0.7967407407, 0.9364907407, 0.9687500000, 0.9364907407, 0.9259259259, 0.8553240741, 0.8720000000, 0.8553240741, 0.8553240741, 0.7741759259, 0.7741759259, 0.8873425926, 0.6989629630, 0.9259259259, 0.9364907407, 0.9142500000, 0.8553240741, 0.6989629630

Comparing the identification results obtained using the "max fuzzy logic operator OR" with that obtained using the "prod. fuzzy logic operator OR" it is easy to see that the difference is significant. These differences follows from the two different meaning of the fuzzy logic operator OR.

Fuzzy logic operator XOR

According to [102] in fuzzy logic, two kinds of XOR operator are used the so called "product fuzzy logic operator XOR" and a so called "min-max fuzzy logic operator XOR".

Product fuzzy logic operator XOR

In case of the fuzzy statements (x is A_1), (x is A_2) the "product fuzzy logic operator XOR" transform this fuzzy statements in the fuzzy statement (x is A_1) XOR (x is A_2) denoted usually with" prod (x is A_1) XOR (x is A_2) "The fuzzy subset $D_{prod-xor}$ which represent the fuzzy statement" prod(x is A_1)XOR(x is A_2) " membership function is $f_{Dprod-xor}(x) = .f_{A1}(x)+ f_{A2}(x)-2f_{A1}(x)*f_{A2}(x)$. This fuzzy subset usually is denoted by $D_{prod-xor}$ = prod ($A_1 \cup A_2$) and is called the "prod fuzzy union" of fuzzy subsets A_1 and A_2. The DOF of x from $D_{prod-xor}$ is given by DOF (prod (x is A_1) XOR (x is A_2)) = $f_{A1}(x)+ f_{A2}(x)-2f_{A1}(x)*f_{A2}(x)$. If A_1 is the fuzzy subset, A_T of intelligence in WAIS scale and A_2 is the fuzzy subset A_{Indeed} of indeed intelligent then their "prod XOR fuzzy union" is presented in the next figure (Figure 15)

The significance of representing the fuzzy statement "prod(x is A_1)XOR(x is A_2)" with fuzzy subset $D_{prod-xor} = A_T \cup A_{Indeed}$ allows the use of the computer to find the DOF of the fuzzy statement prod "(x is intelligent) XOR (x is indeed intelligent)" of persons in case of an IQ data set. Thus, if the IQ data set is:

$IQ :=$ [71, 74, 53, 61, 41, 50, 65, 72, 119, 115, 119, 120, 125, 124, 125, 125, 129, 129, 123, 49, 50, 50, 47, 65, 132, 135, 153, 152, 42, 60, 80, 81, 121, 125, 132];

then using the membership function of $D_{prod-xor} = A_T \cup A_{Indeed}$ the following set of DOFIQ is found :

[0.4989074074, 0.4834074074, 0.2698703704, 0.4235000000, 0.01720370370, 0.2037037037, 0.4745370370, 0.4957037037, 0.3902037037, 0.3125000000, 0.3902037037, 0.4074074074, 0.4745370370, 0.4640000000, 0.4745370370, 0.4745370370, 0.4989074074, 0.4989074074, 0.4519074074, 0.1815000000, 0.2037037037, 0.2037037037, 0.1375370370, 0.4745370370, 0.4957037037, 0.4745370370, 0.1375370370, 0.1594074074, 0.03540740741, 0.03540740741, 0.4074074074, 0.3902037037, 0.4235000000, 0.4745370370, 0.4957037037]:

The above DOFIQ make possible to use

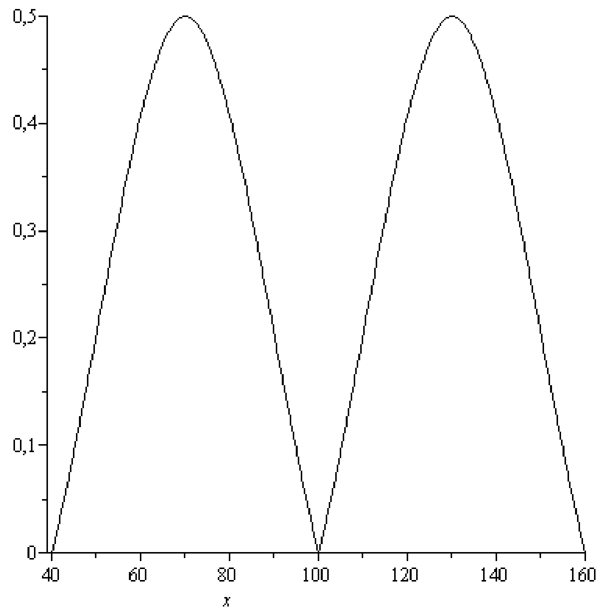

Figure 15 "prod. XOR union" Dprod-xor = $A_T \cap A_{Indeed}$.

computer program for find persons having DOFIQ in a prior given region.

There are 35 persons having DOFIQ in the interval (0, 0.6666666667). Namely:

0.4989074074, 0.4834074074, 0.2698703704, 0.4235000000, 0.0172037037, 0.2037037037, 0.4745370370, 0.4957037037, 0.3902037037, 0.3125000000, 0.3902037037, 0.4074074074, 0.4745370370, 0.4640000000, 0.4745370370, 0.4745370370, 0.4989074074, 0.4989074074, 0.4519074074, 0.1815000000, 0.2037037037, 0.2037037037, 0.1375370370, 0.4745370370, 0.4957037037, 0.4745370370, 0.1375370370, 0.1594074074, 0.0354074074, 0.0354074074, 0.4074074074, 0.3902037037, 0.4235000000, 0.4745370370, 0.4957037037

There are 0 persons having DOFIQ in the interval (0.5,1)

Min-Max fuzzy logic operator XOR

In case of the fuzzy statements (x is A_1), (x is A_2) the "min-max fuzzy logic operator XOR" transform this fuzzy statements in the fuzzy statement (x is A_1)XOR(x is A_2) denoted usually with" min-max(x is A_1)XOR(x is A_2) " The fuzzy subset $C_{min-max-xor}$ which represent the fuzzy statement " min-max(x is A_1)XOR(x is A_2) " membership function is $f_{Cmin-max-xor}(x) = \max\{\min(1-f_{A1}(x),f_{A2}(x)),\min(f_{A1}(x),1-f_{A2}(x))\}$ This fuzzy subset usually is denoted by $C_{min-max-xor}$ = min-max ($A_1 \cup A_2$) and is called the "min-max fuzzy union" of fuzzy subsets A_1 and A_2 . The DOF of x from $C_{min-max-xor}$ is given by DOF(min-max(x is A_1)XOR(x is A_2)) = $\max\{\min(1-f_{A1}(x),f_{A2}(x)),\min(f_{A1}(x),1-f_{A2}(x))\}$. If A_1 is the fuzzy subset, A_T of intelligence in WAIS scale and A_2 is the fuzzy subset A_{Indeed} of indeed intelligent then for the value of IQ x = 71 the following equalities hold: $f_{AT}(x)$ = 0.5166666667; $f_{AIndeed}(x)$= 0.5327777778 ;1- $f_{AT}(x)$ = 0.4833333333;1-$f_{AIndeed}(x)$ = 0.467222222. Therefore, min(1-$f_{AT}(x)$,$f_{AIndeed}(x)$) = min(0.4833333333, 0.5327777778) = 0.4833333333 and

min($f_{AT}(x)$,1-$f_{AIndeed}(x)$) = min(0.5166666667, 0.467222222) = 0.467222222.It follows that DOF(IQ=71) = max{0.4833333333,0.467222222} = 0.4833333333.

Fuzzy logic operator IF...THEN....

According to [102], using the conditional possibility distribution, in fuzzy logic, two type of logic operator IF...THEN... can be defined. The so called "min fuzzy logic operator IF...THEN..." and a so called "product fuzzy logic operator IF... THEN..."

Min fuzzy logic operator "IF ... THEN ..."

Let A a fuzzy subset of universe U and B a fuzzy subset of universe V respectively. In case of the fuzzy statements (u is A), (v is B) the fuzzy logic operator min"IF ... THEN " transform these fuzzy statements in the fuzzy statement IF (u is A)THEN(v is B) denoted usually with" min IF(u is A)THEN(u is B) " The fuzzy subset $C_{min-if-then}$ which represent the fuzzy statement " min IF(u is A)THEN (v is B) " is a subset of the universe UxV and its membership function is $f_{Cmin-if-then}$ = 1 for $f_A(u) < f_B(v)$ and $f_{Cmin-if-then} = f_B(v)$ for $f_A(u) > f_B(v)$ [103]. The DOF of (u,v) from $C_{min-if-then}$ is given by: DOF(min IF(u is A)THEN(v is B)) = 1 for $f_A(u) < f_B(v)$ and DOF(min IF(u is A)THEN(v is B)) = $f_B(v)$ for $f_A(u) > f_B(v)$. If A is the fuzzy subset of very intelligent persons A = A_{Wery} and B is the fuzzy subset of intelligent persons B = A_T then computing the membership function $f_{Cmin\text{-}if\ then}$ the following result is found: for $40 < u < 100$ and $40 < 200/3 + u^2/60 - 4u/3 < v < 100$ $f_{Cmin\text{-}if\ then}(u,v) = 1$; for $40 < u <$,100 and $40 < v < 200/3 + u^2/60 - 4u/3 < 100$ $f_{Cmin\text{-}if\ then}(u,v) = (v-40)/60$;for $100 < u < 160$ and $100 < v < 160$ $f_{Cmin\text{-}if\ then}(u,v) = -(v-160)/60$. Therefore: For u = 60 and v = 70 $f_{Cmin\text{-}if\ then}(u,v) = 1$; for u = 95 and v = 50 $f_{Cmin\text{-}if\ then}(u,v) = 24/121$; for u = 110 and v = 130

$f_{Cmin\text{-}if\ then}(u,v) = 18/25$; for u = 155 and v = 120 $f_{Cmin\text{-}if\ then}(u,v) = 1$.

Prod. fuzzy logic operator "IF … THEN …"

Let A be a fuzzy subset of universe U and B a fuzzy subset of universe V respectively. In case of the fuzzy statements (u is A), (v is B) the fuzzy logic operator prod "IF … THEN …" transform these fuzzy statements in the fuzzy statement IF (u is A) THEN (v is B) denoted usually with" prod IF (u is A) THEN (v is B) "The fuzzy subset $C_{prod\text{-}if\text{-}then}$ which represent the fuzzy statement" prod IF (u is A) THEN (v is B) "is a subset of the universe UxV and its membership function is $f_{Cprod\text{-}if\text{-}then} = 1$ for $f_A(u) = 0$ and

$f_{Cprod\text{-}if\text{-}then} = \min\{1, f_B(v)/f_A(u)$ for $f_A(u) = 0\}$.[103] The DOF of (u,v) from $C_{prod\text{-}if\text{-}then}$ is given by: DOF(prod IF(u is A)THEN(v is B)) = 1 for $f_A(u) = 0$ and DOF(prod IF(u is A)THEN(v is B)) = $\min\{1, f_B(v)/f_A(u)$ for $f_A(u) > 0\}$. If A is the fuzzy subset of very intelligent persons $A = A_{Wery}$ and B is the fuzzy subset of intelligent persons B = A_T then computing the membership function $f_{Cprod\text{-}if\ then}$ the following result is found :If u = 40 or u = 160 then $f_{Cprod\text{-}if\ then}(u,v) = 1$; if $40 < u < 100$ and $40 < v < 100$ then $f_{Cprod\text{-}if\ then}(u,v) = [(v-40)/60]/[(u-40)/60]^2$ for $[(v-40)/60]-[(u-40)/60]^2 < 0$ and $f_{Cprod\text{-}if\ then}(u,v) = 1$,for $[(v-40)/60]-[(u-40)/60]^2 > 0$; if $100 < u < 160$ and $100 < v < 160$ then $f_{Cprod\text{-}if\text{-}then}(u,v) = [-(v-160)/60]/[-(u-160)/60]^2$ for $[-(v-160)/60]-[-(u-160)/60]^2 < 0$ and

$f_{Cprod\text{-}if\ then}(u,v) = 1$,for) = $[-(v-160)/60]/[-(u-160)/60]^2 > 0$. Therefore: for u = 60 and v = 70 $f_{Cprod\text{-}if\ then}(u,v) = 1$; for u = 95 and v = 50 $f_{Cprod\text{-}if\ then}(u,v) = 24/121$; for u = 155 and v = 120

$f_{Cprod\text{-}if\ then}(u,v) = 1$; for u=110 and v=130 $f_{Cprod\text{-}if\ then}(u,v) = 18/25$.

Results

For the interpretation of IQ tests to the human intelligence a triangular fuzzy set is associated. By using the membership function this fuzzy set, to an IQ value a degree of confidence of the IQ value is associated. This operation is made also in case of linguistic modifiers (Kind of understanding) of the word human intelligence i.e.: very intelligent, more or less intelligent, indeed intelligent, etc. In this way the same IQ value has different confidence degree in different interpretation of the word intelligent. These differences are illustrated in case of a given set of IQ values showing that the differences can be significant in case of identification of group of persons having IQ in a given range and degree of fulfillment in the corresponding range. In this way computationally become possible to compare different kind of interpretations of the verbal expression human intelligence via degree of fulfillment. Moreover, the use of fuzzy logic operators is illustrated by using the same set of IQ values.

Authors Contribution

The authors contributed equally to the realization of this work. All authors have read and agreed to the published version of the manuscript.

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The original contributions presented in the study are included in the article; further inquiries can be directed to the corresponding author.

Conflicts of Interest

The authors declare no conflicts of interest.

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