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SHORT COMMUNICATION

The Conservation Laws in Quantum Mechanics

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ABSTRACT

Is often claimed that Einstein is wrong about quantum mechanics. However, when compared with respect to theoretical foundations rather than experimental results Einstein's theories are found to be superior. Although quantum mechanics correctly predicts what it is possible to observe (the emissions) it ignores the other half of natural phenomena, what cannot be observed (the absorptions), thereby violating the conservation laws. By describing only one-half of quantum mechanics conceptual difficulties such as wave function collapse, infinite paths, and inscrutable mathematics seem to appear out of nowhere. The deficiencies are corrected by introducing Hamilton's principle S = ∫ Ldt and applying the methods of quantum field theory to derive relativistic equations of motion. The calculus of variations is used to show that the wave function represents an incomplete equation of motion because it equals twice the allowable action minimum.

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Introduction

It is often claimed, whether overtly or implied, that the most accurate theory is the best theory. A clock that was accurate to within one second over the age of the universe was later improved upon by a clock accurate to within 100 milliseconds. The most recent clock experiment can differentiate between gravitational potentials of one millimeter [1]. Clock accuracy is continuously being improved upon because it is believed that more accurate time measurement "offers new opportunities for tests of fundamental physics". In another area of research universities are competing to improve upon measurements of the g-2 factor of an electron [2]. Once again the goal is to find a discrepancy with the standard model that will lead to "new physics". Improved instrumentation, more accurate measurements, and better predictions are the procedures that many say will lead to an improved understanding of Nature. We will show in these pages that improved accuracy does not equate with improved understanding and that it is precisely what cannot be observed that is the key to understanding natural phenomena.

The importance of the observer, and by inference the observation, in scientific inquiry is a sticking point that has a long history in theoretical physics. On one side is Bohr and the majority of physicists. "It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature." On the other side of the dispute, Einstein believed that, "On principle, it is quite wrong to try founding a theory on observable magnitudes alone." Our purpose is to determine which of these

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two approaches is the more accurate, the empirical or the intuitive.

Momentum Conservation

Einstein's theory

Einstein's first attempt at a theory of quantum mechanics was a derivation of Planck's law by statistical methods "in an amazingly simple and general manner" [3]. There he describes the dynamic equilibrium that exists between the thermal energy absorbed by molecules and its subsequent quantum mechanical emission as black body radiation. Heat energy that is absorbed according to classically defined Maxwell Boltzmann statistics is transformed at the molecular level and emitted according to the Planck radiation law. The absorption of energy by a molecule leads to momentum +E/c in the direction of propagation, while the *emission* of a photon causes a recoil momentum -E/c that is directed in the opposite direction of propagation. The sudden reversals and random nature of the impulses cause molecular trajectories to be discontinuous, as observed in Brownian motion.

As is typical of Einstein's work he begins his derivation at a fundamental level taking into consideration the conservation laws. "If a radiation beam with a well-defined direction does work on a Planck resonator [quantum oscillator], the corresponding energy is taken from the beam. According to the law of conservation of momentum, this energy transfer corresponds also to a momentum transfer from the beam to the resonator." He demonstrates here the close association that exists between momentum and energy. Molecular impulses transfer energy to the oscillator which then radiates the energy when an electron decays. Thus energy absorption is differentiated from energy emission. He continues, "Let a molecule of given kind be in uniform motion with speed **v** along the X-axis of the coordinate system K. We inquire about the momentum transferred on the average from the radiation to the molecule per unit time. To calculate this we must consider the radiation from a coordinate system K′ that is at rest with respect to the given molecule. For we have formulated our hypotheses about emission and absorption only for molecules at rest." The molecules in a gas absorb energy classically in K and emit energy quantum mechanically in K′. The absorbed energy must be equal to the emitted energy due to the conservation of energy.

The emission and absorption of energy is to be described quantum mechanically in the coordinate system K' which is "at rest with respect to the given molecule", while the kinetic energy of molecules will be treated with "ordinary mechanics" in the coordinates of the system K. To determine the total energy of a molecule we sum a classical component due to kinetic energy, as determined by temperature; and a quantum mechanical component due to the energy of excited states. The inclusion of both classical and quantum mechanical energy forms in the same model of radiation is a feature that distinguishes Einstein's methods from all others. It contrasts sharply with the Bohr-Heisenberg method, which derives two independent expressions, one classical and one quantum mechanical, and then links them by using the correspondence principle. The Schrödinger method links the quantum and classical worlds by means of wave function collapse, a process that introduces conceptual difficulties and violates special relativity theory. However, the quantum classical divide was never a problem for Einstein, for he accepted it as a fundamental property of matter. There are classical laws governing molecular behavior in K and quantum laws governing a molecule's behavior in K'; two points of view of a single reality.

Heisenberg's theory

Matrix mechanics evolved from long-standing attempts to describe dispersion phenomena, the continuous change in the angle of refraction of different frequencies of light by a prism or other medium. Although light disperses continuously across the entire spectrum, at certain specific frequencies characteristic of the medium, it is completely absorbed forming lines. When complex sets of mathematical rules were discovered that describe the relationship between the observed frequencies and intensities of spectral lines it allowed Heisenberg to formulate a theory of quantum mechanics that reconciles the continuity of radiation fields with the discrete energy states of an atom by expressing electron transitions in the form of a matrix [4].

$$\sum_k \left(p_{nk}q_{km} - q_{nk}p_{km}\right) = \begin{matrix} i\hbar \text{for } n = m \\ 0 \text{for } n \neq m \end{matrix} \quad (1)$$

Although p in the above equation refers to momentum it is not the momentum of a molecule in K used by Einstein, rather it is the momentum due to photon emission in K'. To compare 1) more closely with Einstein's statistically defined measurements of

energy fluctuation we refer to the energy matrix [5]. The diagonal elements of the matrix $m = n$ refer to all possible energy states of a quantum system. Because the theory only concerns quantum mechanical phenomena it represents a complete break from classical theory. Its weakness lies in the fact that it does not differentiate between how an atom absorbs energy and how energy is emitted. In other words, equation 1) is formulated exclusively in coordinates relative to K'.

Off-diagonal elements of the energy matrix $m \neq n$ refer to the resonances of radiation with an atom's valence electrons, where elements of the array E_{mn} that are above the diagonal have a plus sign because they represent energy absorption and elements of the array that are below the diagonal E_{nm} have a minus sign representing energy emission. Because the energy of an absorption offsets the energy of an emission except for a difference in phase a value of zero is assigned to these matrix elements when averaged over time. However changes in state do not account for all contributions of energy when examined microscopically. Spectral lines also broaden due to exchanges of momentum caused by thermal energy that cannot be avoided. Although the collisions cannot be observed individually they are evident statistically in measurements of gas temperature. Heisenberg did not take these microscopic interactions into account because he believed that quantum mechanics should be "founded exclusively upon relationships between quantities which in principle are observable" [4]. This caused him to focus exclusively upon the intensity and frequency of the spectral lines, which are the emission properties of a radiating atom or molecule.

Due to the conservation of energy an atom must first absorb energy from surrounding atoms before it can emit energy. Einstein realized the importance of unobserved momentum exchanges stating [3], "In general one is satisfied with a consideration of the *energy* exchange without consideration of the exchange of momentum. One feels easily justified to do so, because the smallness of the momenta transferred by radiation are almost always negligible when compared to other motion-generating causes. But in *theoretical* investigations these small effects are definitely as important as the more prominently appearing energy transfers by radiation, because energy and momenta are always intimately linked together." What Einstein did not say was that energy and momentum are linked together by the conservation laws. Thermal energy absorbed by oscillators during black body radiation due to momentum exchange is subsequently emitted quantum mechanically as radiation. If the classically defined absorption energy in K is discounted because it is unobservable or too small to measure one-half of all radiation processes are arbitrarily eliminated.

Feynman's theory

In the path integral approach to quantum mechanics we can see violations of the conservation laws even more clearly. In the following passage Feynman considers absorption and emission by first using the field approach of classical theory and then that of quantum field theory, but he then rejects both in favor of a particle model of only emission [6]."If one solves the problem of an atom being perturbed by a potential varying sinusoidally with time, which would be the situation if matter were quantum mechanical and light classical, one finds indeed that it will in all probability eject an electron whose energy shows an increase of $h\nu$, where ν is the frequency of variation of the potential. When, however, we come to spontaneous emission and the mechanism of the production of light, we come much nearer to the real reason for the apparent necessity of photons. The fact that an atom emits spontaneously at all is impossible to explain by the simple picture given above. In empty space an atom emits light and yet there is no potential to perturb the systems and so for it to make a transition. The explanation of modern quantum mechanical electrodynamics is that the atom is perturbed by the zero-point fluctuations of the quantized radiation field. It is here that the theory of action at a distance gives us a different viewpoint. It says that an atom alone in empty space would, in fact, not radiate. Radiation is a consequence of the interaction with other atoms (namely, those in the matter which absorbs the radiation). We are then led to the possibility that the spontaneous radiation of an atom in quantum mechanics also, may not be spontaneous at all, but induced by the interaction with other atoms, and that all of the apparent quantum properties of light and the existence of photons may be nothing more than the result of matter interacting with matter directly and according to quantum mechanical laws."

Due to considerations of energy conservation Feynman's discussion concerning radiation processes begins accurately; that is, atoms absorb energy before radiating and an atom in empty space would not radiate at all. However, that realization

did not stimulate thoughts about how to incorporate absorption into a *complete* theory of radiation, rather it caused him to formulate an emission theory as a sum over paths [7]. By deriving a theory of only emissions he obtains extremely accurate predictions, but at the expense of mathematical consistency. Spurious infinities associated with the self-energy of an electron must be artificially subtracted away using "renormalization" before meaningful calculations and realistic predictions can be made. An expected result of the failure to maintain a balance between absorption and emission energies are particle paths unbounded both in geometry, by curved and looping trajectories, and spatially, by paths that exceed the energy of excited states and can even extend to infinity. Both are clear violations of the conservation laws and of special relativity theory.

Relativistic Quantum Mechanics

Energy absorption

A theory of quantum mechanics compatible with special relativity is sought after by describing radiation processes with the time integral of a Lagrangian and applying the calculus of variations. Consider a radiating atom with ground state |1> and excited state |2>. To describe the electron's equation of motion during energy absorption we make use of Hamilton's principle function S = ∫ Ldt, where L=T-V. It provides for a more economical expression of the laws of motion by specifying fixed boundary conditions for particle paths rather than trajectories in the Cartesian coordinates of Newton's laws. The limitations of absolute space and time are thereby avoided.

The absorption of energy by an atom may occur discretely as observed in the photoelectric effect; however, it more often occurs continuously due to the superposition of transverse fields as in the case of optical phenomena. The fields are unbounded so to describe energy absorption we use the particle model and describe the electron during a continuous excitation. Let the electron initiate its motion at a fixed point on the equipotential surface R_1 of the ground state at time t_1, proceed along a path r, and upon arriving at the excited state R_2 adopt the circular path $2\pi r$ of an orbital thereby assuming orbital angular momentum. The action minimum S[r(t)] for a path between the stationary points R_1 and R_2 yields not zero as in classical dynamics, but the reduced Planck's constant ħ due to angular momentum.

$$S\left[r(t)\right] = \int_{R_1}^{R_2}\int_{t_1}^{t_2} L dt = \hbar \qquad (2)$$

The action S is a functional, S[r(t)], which means it has as its argument an infinite number of functions, the possible electron trajectories r(t). If we arbitrarily set the initial time $t_1(r)$ at zero, then the action integral for a path of unit length is evaluated as follows:

$$E_{12}\,\tau = \hbar$$

The electron's path is described in a configuration space consisting of three coordinates that define the origin, or nucleus, and three coordinates that define the manifolds R_1 and R_2 upon which the electron is constrained to move; that is, the electron shells. In other words, the quantum oscillator is described in configuration space as having six parameters, three for the nucleus and three for the electron shells, a total of six degrees of freedom.

Energy emission

At the relativistic or high end of the energy spectrum in quantum field theory, particles are treated as excited states of the more fundamental underlying quantum fields. As Nobel laureate Frank Wilczek noted [8], "In quantum field theory, the primary elements of reality are not individual particles, but underlying fields." On the other hand, when we examine lower energy interactions in nonrelativistic theory we find that interpretations are almost exclusively about particles. It seems that theoreticians use fields when it is convenient to use fields, and particles when it is convenient to use particles. However, the physical principles that underlie mathematical models demand a level of consistency that goes beyond simple expediency.

In quantum field theory particles are described by an action functional $S[\psi(x_i)]$ where the fields $\psi(x_i)$ of particles are defined throughout space. Each of the elementary particles has a field and the action depends on all of the fields. The equations of motion are determined by minimizing the Lagrangian in a region of space-time. We will follow these same practices in order to extend the field interpretation to nonrelativistic theory. The region of space-time that is of interest for lower energy interactions lies between the two states of an electron transition. Therefore we specify a field boundary coincident with the atom's excited state, an electron shell, that acts to

localize particles and fields inside the atomic space. Within the space-time region we define a Lagrangian density of the fields and their first derivatives $\pounds(\phi_i, \phi_{i,\mu})$ which allows for a complete accounting of the energy interactions, where ϕ_i is the current density and $\phi_{i,\mu}$ is the electromagnetic field strength. The action integral for a quantum oscillator with an outer electron that occupies either of two allowable energy states may now be formulated in a way that is consistent with special relativity theory, where emission initiates from the excited state $R_2 = (x_2,y_2,z_2)$ at time t_2 and it finalizes at the ground state $R_1 = (x_1,y_1,z_1)$ at time t_1. Applying Hamilton's principle we require the integral of the Lagrangian density over the region of space-time between the excited and ground states to be a minimum for all small variations of the coordinates inside the region, where the action minimum for an arbitrary quantum system is defined in angular measure to be the reduced Planck's constant ħ.

$$S\left[\Phi_i(t)\right] = \int_{R_2}^{R_1}\int_{t_2}^{t_1} \pounds\left(\Phi_i \Phi_{i,}\right) d^3x dt = \hbar \qquad (3)$$

The energy states |2> and |1> coincide with the electron shells and determine invariant field boundaries where fields vanish. The change in action yields a relativistic formulation of emission that is invariant, the same for all observers. The action $S[\phi_i(t)]$ is a functional, a function of the values of coordinates on the *discrete* boundaries of the space-time surfaces R_2 and R_1 which are in turn functions of the *continuous* space-time variables of the fields within the surface. The field boundaries are uniquely fixed in four-dimensions by the volume d^3x and the time interval t_2-t_1 causing photon emission to be described as a four-dimensional localization of fields.

Comparison of the relativistic and non-relativistic models

The overriding problem in quantum mechanics historically has been how to describe two physical processes, absorption and emission, with a single equation. According to the Schrödinger wave equation they occur as a single process that evolves symmetrically in time. The wave function ψ used to describe an electron oscillating between two energy states performs two complete rotations, or a total of 720 degrees, before returning to its original state. If the electron is represented mathematically by a vector in Hilbert space |ψ> then one rotation of 2π results in a negative value -|ψ> and a second rotation of 2π brings the electron back to its original state |ψ>. The rotation occurs in abstract space so no physical interpretation is possible.

Models of quantum systems that use action functionals describe quantization in real space and real time as four-dimensional absorptions of energy by the electron 2), followed by four-dimensional localizations of field 3) and release of a photon. We interpret the first rotation of the wave function not as a rotation in abstract space, but as the change in phase of electromagnetic fields from 0 to 2π in real space during absorption. Thus one "rotation" of the wave function is interpreted as one full cycle of an electromagnetic wave and an increase in the electron's energy from the ground state to an excited state. The second rotation occurs as the electron returns to the ground state and is interpreted as a localization of electromagnetic field energy and emission of a photon. The dual wave-particle nature of the photon is thereby realized in a physical transformation.

The external appearances of a radiating atomic system, the frequency and intensity of its spectral lines, are observables described by the matrix mechanical formulation in 1), where each matrix includes both absorption and emission processes for all possible radiation processes. This is because, due to the conservation of energy, an atom in empty space cannot emit radiation; that is, no observable can be realized unless absorption and emission are both present. A quantum system that is only described by observables, as is true of matrices, is incomplete. Hermitian matrices are specifically chosen to represent quantum mechanical observables because it is a complex square matrix that is equal to its own conjugate transpose. As a result when upper elements, absorptions, are multiplied with lower elements, emissions, real values, eigenvalues, are obtained on the diagonals. Thus a single matrix describes two physical events and noncommutation is the result of different values for the angular momentum of an atomic oscillator, and is equal to ħ in |2> and zero in |1> [9].

Discussion

To determine the true evolution of a quantum system we use Hamilton's principle and take advantage of the symmetries of the system, the generalized coordinates which describe R_2 and R_1. Then the path the electron follows between the

initial and final stationary points of a complete cycle between t_1 and t_2 is the one which minimizes the action. However, the true path is not necessarily the one that is the most precisely determinable. Thus the Schrödinger equation is the most accurate differential equation of motion currently available, but it does not minimize the action. As noted in the previous section the wave function includes twice the minimum allowable action for a quantum oscillator. Therefore it is an incomplete description of atomic structure. Similarly, in the case of matrix mechanics the action of each of the diagonal elements of an infinite array is equal to ħ so the action is not a minimum, it is infinite. Our analysis of 3.1 and 3.2 shows that the field interpretation of quantum field theory is the one that provides the true evolution of a quantum system.

Conclusion

If Einstein was wrong about certain aspects of quantum mechanics, it was because he did not understand the abstruse nature of the mathematics; something anyone who has ever studied quantum mechanics can relate to. By failing to understand the other quantum theories, especially matrix mechanics, he was unable to point out their imperfections. Nevertheless the foundations of his own quantum theory are crystal clear and can be used to evaluate other theories, all of which claim to describe the same radiation processes. One of the insights he had, which is the topic of this discussion, was that every quantum system has two facets and needs two equations to describe it. Thus the quantum-classical divide is not a mystery that has to be independently solved by "wave function collapse", rather it is fundamental to all observable material systems.

Each of the three formulations of non-relativistic quantum mechanics provides a unique perspective to atomic structure by emphasizing a different physical aspect of the three constituent field sources of a radiating quantum oscillator; electron, photon, and nucleus. This may be compared to the simpler three-dimensional practice in architecture of providing three visual perspectives to a building. Each one provides a partial view, and when taken together they give an improved understanding of the structure as a whole. The "whole" of quantum mechanics is given of course by the action functionals.

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